



A virtual laboratory system for quantum mechanics education

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Abstract

With the appearance of quantum theory, more and more people began to focus on this area, but it is so confusing that its education is very difficult. First, the article discussed a virtual laboratory system for quantum mechanics education. Second, the rule setting is also analyzed to provide us an effective proof system and correctness system to affirm virtual quantum laboratory. Third, And virtual laboratory is studied from its operational system, rule setting and semantics environment which includes the input and output of virtual laboratory. Last, some conclusions and future directions are given at the end of the paper.

Keywords: virtual laboratory, computer aided instruction, education system, quantum mechanics

1. Introduction

1.1 Related Work

Today, virtual laboratory plays a very important role in modern education. Oke, *et al* (2016) ^[1] introduced a Marine Virtual Laboratory (version 2.1) enabling efficient ocean model configuration. Al-Khalifa (2017) ^[2] presented CHEMOTION: A gesture based chemistry virtual laboratory with leap motion. Dehn (2017) ^[3] described Is it a construction material from the virtual Laboratory. Ortega-Moody, *et al* (2017) ^[4] proposed Virtual laboratory of industrial scenarios for training in the areas of automation and control. Popovic, Naumovic (2016) ^[5] studied virtual laboratory and learning management system in optimal control theory education. Jezierska, *et al* (2016) produced the virtual laboratory for student understanding of biophysics. Gomez-Sacristan (2016) ^[7] talked about virtual laboratory for QoS study in next-generation networks with metro ethernet access. Liu (2017) ^[8] discussed the design and implementation of intelligent computer aided instruction system based on WEB. Xiao (2017) ^[9] displayed research on the construction of computer aided instruction system based on comprehensive education model. Siqueira, (2017) ^[10] introduced a specific segment of the private education system: low-cost private schools, Portillo-Torres (2017) ^[11] made a research on education: prospects and challenges for the education system. Sergeev (2017) ^[12] presented onward to international free education system in dermatology. Kobayashi, *et al* (2017) ^[13] described an application framework for smart education system based on mobile and cloud systems.

Recently quantum mechanics appeared and presented us a difficult in its education because it is so confusing. Ryoji (2016) exhibited inequality of effort in an egalitarian education system. Abramo, (2016) ^[15] put forward The north-south divide in the Italian higher education system. Lopez (2016) ^[16] illustrated Preliminary analysis of farm workers perception about continuous education system in the Region of Murcia. Dutta (2015) ^[17] developed Some drawbacks of the

higher education system in India. Gong, Liu, Jiao, *et al* (2015) ^[18] put forward a novel earthquake education system based on virtual reality. Zhang, (2017) ^[19] proposed what regulates the catalytic activities in AGE catalysis, and an answer from quantum mechanics/molecular mechanics simulations. Alhaidari (2017) ^[20] discussed reconstructing the potential function in a formulation of quantum mechanics based on orthogonal polynomials. Aerts, (2017) ^[21] researched the extended bloch representation of entanglement and measurement in quantum mechanics. Fernandez, *et al* (2015) ^[22] extended Ad Hoc physical Hilbert spaces in quantum mechanics.

1.2 Organization of the Article

Section 2 makes the definition of the operational system of virtual laboratory. Section 3 includes the rule setting of virtual laboratory. In section 4, the initiation of semantics environment and the input and output of virtual laboratory is described. In section 5, it includes the verification for virtual quantum laboratory. In section 6, examples and illustration are made descriptions of. Section 7 is the conclusions of the full text.

2. Operational system of virtual laboratory

Here H_{all} is written for the tensor production of these situation spaces of any quantum variable; that is,

$$H_{all} = \bigotimes_{all \ x} H_x$$

For simple presentation, E is applied to stand for the void program. A quantum arrangement is a set (B, p) , in which B is a quantum programme or $E, p \in \Gamma^-(H_{all})$ is a part denseness performer upon H_{all} , and it is applied to denote the (global) situation of quantum variable.

Let $\bar{x} = x_1, \dots, x_n$ be a quantum record. A linear performer

γ on $H_{\bar{x}}$ contains a cylinder expansion

$$\gamma \otimes U_{Var\{\bar{x}\}} \quad (1)$$

on H_{all} , in which $U_{Var\{\bar{x}\}}$ is the unchanged performer upon the Hilbert space $\bigotimes_{x \in Var\{\bar{x}\}} H_x$

As a result, γ is here for its expansion Eq. (1), and it could be known without any difficulty from the framework, without being confused.

This performing semantics of quantum programme is made definition of as a transitional connection \rightarrow between quantum arrangement. A transition

$$\langle B, p \rangle \rightarrow \langle B', p' \rangle$$

denotes that after performing quantum programme B , one procedure in situation p , the situation of quantum variable gets p' , and B' is the remain of B yet to performed. Particularly, if $B' \equiv E$, then B ends in situation p' . The initializing “ $x := 0$ ” arranges quantum variable x to the fundamental situation $|0\rangle$. In the law (Initializing) and as a result, $|\phi\rangle_x$ is applied to denote the situation $|\phi\rangle$ of the system indicated by quantum variable x . Therefore, for any situations $|\phi\rangle$, $|\psi\rangle$ of the x system, an performer $|\phi\rangle_x \langle \psi|$ can be made definition of on the situation space.

The x system, as what is listed below:

$$\left(|\phi\rangle_x \langle \psi| \right) \left(|\phi'\rangle_x \right) = \langle \psi | \phi' \rangle \left(|\phi\rangle_x \right)$$

for each situation $|\phi'\rangle$ of the x system. At times, $|\phi\rangle_x \langle \psi|$ ought to be comprehended as its cylinder expansion on H_{all} on the basis of Eq. (1). Similarly, $|\phi\rangle_{\bar{x}}$ and $|\phi\rangle_{\bar{x}} \langle \psi|$ can be made definition of while \bar{x} is a quantum record and $|\phi\rangle$ and $|\psi\rangle$ are situations in $H_{\bar{x}}$. In order to see the part of initializing more apparently, the situation of $type(x) = \text{integer}$ is considered to be an example. Firstly, a special situation is reviewed in which p is a pure situation; that is, $p = |\psi\rangle \langle \psi|$ for certain $|\psi\rangle \in H_{all}$. $|\psi\rangle$ is written in this type:

$$|\psi\rangle = \sum_k g_k |\psi_k\rangle,$$

In which $|\psi_k\rangle$ is a production situation for each k , for example

$$|\psi_k\rangle = \bigotimes_{all\ x'} |\psi_{kx'}\rangle$$

Then

$$p = \sum_{k,l} g_k g_l^* |\psi_k\rangle \langle \psi_l|,$$

After the initializing the situation getss:

$$\begin{aligned} p_0^x &= \sum_{n=-\infty}^{\infty} |0\rangle_x \langle n| p |n\rangle_x \langle 0| \\ &= \sum_{k,l} g_k g_l^* \left(\sum_{n=-\infty}^{\infty} |0\rangle_x \langle n| \psi_k \rangle \langle \psi_l | n \rangle_x \langle 0| \right) \\ &= \sum_{k,l} g_k g_l^* \left(\sum_{n=-\infty}^{\infty} \langle \psi_{lx} | n \rangle \langle n | \psi_{kx} \rangle \right) \left(|0\rangle_x \langle 0| \otimes_{x' \neq x} |\psi_{kx'}\rangle \langle \psi_{lx'}| \right) \quad (2) \\ &= \sum_{k,l} g_k g_l^* \langle \psi_{lx} | \psi_{kx} \rangle \left(|0\rangle_x \langle 0| \otimes_{x' \neq x} |\psi_{kx'}\rangle \langle \psi_{lx'}| \right) \\ &= |0\rangle_x \langle 0| \otimes \left(\sum_{k,l} g_k g_l^* \langle \psi_{lx} | \psi_{kx} \rangle \otimes_{x' \neq x} |\psi_{kx'}\rangle \langle \psi_{lx'}| \right) \end{aligned}$$

Generally, assume p is created through a whole $\{(\zeta_i, |\psi_i\rangle)\}$ of pure situations; that is,

$$p = \sum_i \zeta_i |\psi_i\rangle \langle \psi_i|.$$

For every i , $p_i = |\psi_i\rangle \langle \psi_i|$ is written down and it is supposed that it gets p_{i0}^x after the initializing. Through the above argumentation, p_{i0} can be written down in the following type mentioned next:

$$p_{i0}^x = \sum_k g_{ik} \left(|0\rangle_x \langle 0| \otimes |\phi_{ik}\rangle \langle \phi_{ik}| \right),$$

In which $|\phi_{ik}\rangle \in H_{Var\{x\}}$ for all k . Then the initializing makes the p get

$$\begin{aligned} p_0^x &= \sum_{n=-\infty}^{\infty} |0\rangle_x \langle n| p |n\rangle_x \langle 0| \\ &= \sum_i \zeta_i \left(\sum_{n=-\infty}^{\infty} |0\rangle_x \langle n| p_i |n\rangle_x \langle 0| \right) \quad (3) \\ &= \sum_{i,k} \zeta_i g_{ik} \left(|0\rangle_x \langle 0| \otimes |\phi_{ik}\rangle \langle \phi_{ik}| \right) \end{aligned}$$

From Eqs. (2) and (3) it can be seen that the situation of

variable x is arranged to be $|0\rangle$ and situations of the alternative quantum variable are invariant. The statement “ $\bar{x} := \delta\bar{x}$ ” simply indicates that single transformation δ is operated on quantum record \bar{x} , leaving the situations of the quantum variable not in \bar{x} invariant. Notice that δ in the targeted arrangement of the law (Single Transformation) indeed represents the cylinder expansion of δ on H_{all} (see Eq.(1)). A resembling remark is used to the laws for measuring and loops. Sequential work resembles its opposite number in classical computation. The programme build “measure $\eta[\bar{x}] : \bar{B}$ ” is a quantum summary of a classical situational statement. Recollect that the initial step in the performance of the situational statement “if C then B_1 else B_2 ” is to examine whether Boolean statement C is met.

3. Rule setting of virtual laboratory

Suppose a countably unlimited arrangement Var of quantum variable. The symbols $x, x', x'', x_0, x_1, x_2, \dots$ would be applied as metavariable varying over quantum variable. Recollect that in classic computation, a type is used to indicate the scope of a variable. Therefore, in quantum computation, a form ought to be the situation space of the quantum system indicated by certain quantum variable. Formally, a form Ω is a title of a Hilbert space H_Ω . In this composition, merely two fundamental forms are considered: Boolean and integer. The outcomes acquired in this composition can be generalized without difficulty to the example with more forms. The Hilbert spaces indicated through Boolean and integer are what listed below:

$$H_{\text{Boolean}} = H_2, \\ H_{\text{integer}} = H_\infty.$$

Notice that the arrangements indicated by forms Boolean and integer in classic computation are just the computational basis of H_{Boolean} and H_{integer} , each. Now suppose that every quantum variable x possess a type $type(x)$, which is Boolean or integer. The situation space H_x of a quantum variable x is the Hilbert space indicated through its type; that is,

$$H_x = H_{type(x)}.$$

A quantum record is made definition of to be a limited order of apparent quantum variable.

The situation space of a quantum record $\bar{x} = x_1, \dots, x_n$ is the tensor production of the situation spaces of the quantum variable taking place in x ; that

$$\text{is, } H_{\bar{x}} = \bigotimes_{i=1}^n H_{x_i}$$

At present the syntax of quantum programmes can be made definition of. The quantum programmes thought in this composition are quantum expansions of classic while-programmes. Formally, they are produced by the grammar

listed below:

$$B ::= \text{skip} \mid x := 0 \mid \bar{x} := \delta\bar{x} \mid B_1; B_2 \mid \text{measure } \eta[\bar{x}] : \bar{B} \\ \bar{B} \mid \text{while } \eta[\bar{x}] = 1 \text{ do } B$$

in which

- x is a quantum variable and \bar{x} a quantum record;
- δ in the expression “ $\bar{x} := \delta\bar{x}$ ” is a single performer on $H_{\bar{x}}$. Particularly, if $type(x) = \text{integer}$, then the expression $x := \delta + kx$, in which $\delta + k$ is the k -translation performer, will usually be short to $x := x + k$;
- in the expression “measure $\eta[\bar{x}] : \bar{B}$ ”, $\eta = \{\eta_m\}$ is a measuring on the situation space $H_{\bar{x}}$ of x , and $B = \{B_m\}$ is a series of quantum programmes such that every result m of measurement η accords with B_m ;
- $\eta = \{\eta_0, \eta_1, \dots\}$ in the expression “while $\eta[\bar{x}] = 1$ do B ” is a yes-no measuring on H_x .

The instinctive idea of these quantum programme builds will get clear after bringing into their performing semantics in the next part.

“ \equiv ” is used to indicate syntactic equilization of quantum programmes. The technical definition listed below will be needed as a result.

Definition 3.1. The arrangement $var(B)$ of quantum variable in quantum programme B is recursively mae definition of like this:

- (1) If $B \equiv \text{skip}$, then $var(B) = \emptyset$;
- (2) If $B \equiv x := 0$, then $var(B) = \{x\}$;
- (3) If $B \equiv \bar{x} := \delta\bar{x}$, then $var(B) = \{\bar{x}\}$;
- (4) If $B \equiv B_1; B_2$, then $var(B) = var(B_1) \cup var(B_2)$;
- (5) If $B \equiv \text{measure } \eta[\bar{x}] : \bar{B}$, then $var(B) = \{\bar{x}\} \cup \bigcup_m var(B_m)$;
- (6) If $B \equiv \text{while } \eta[\bar{x}] = 1 \text{ do } B$, then $var(B) = \{\bar{x}\} \cup var(B)$;

4. Semantics environment of virtual laboratory

The connotational semantics of the quantum programme is made definition of as a semantic operation which charts part denseness performers to themselves. More accurately, for any quantum programme B , the semantic operation of B summarizes the computed outcomes of all ending computations of B .

It is written \rightarrow^* for these reflexive and transitive closing of

$$\rightarrow; \text{ that is } \langle B, p \rangle \rightarrow^* \langle B', p' \rangle$$

if and only if $\langle B, p \rangle \rightarrow^n \langle B', p' \rangle$ for certain $n \geq 0$.

Definition 5.1. Let B be a quantum programme. Then its

semantic operation is made definition of through

$$\llbracket B \rrbracket : \Gamma^-(H_{all}) \rightarrow \Gamma^-(H_{all})$$

for all $p \in \Gamma^-(H_{all})$.

$$\llbracket B \rrbracket(p) = \sum \left\{ p' : \langle B, p \rangle \rightarrow^* \langle E, p' \rangle \right\} \quad (4)$$

It ought to be indicated that $\{\cdot\}$ in Eq. (4) represents a multiset. The reason for applying the multiset is that the identical denseness performer may be acquired by distinguishing computational ways, as it can be seen from the measuring and circle laws in the performing semantics.

Now certain fundamental properties of semantic operations will be established. Firstly, their linearity

LEMMA 5.1 is proved. Let $p_1, p_2 \in \Gamma^-(\Gamma)$ and $\lambda_1, \lambda_2 \geq 0$. If $\lambda_1 p_1 + \lambda_2 p_2 \in \Gamma^-(\Gamma)$, then for any quantum programme B , there is $\llbracket B \rrbracket(\lambda_2 p_2 + \lambda_1 p_1) = \lambda_2 \llbracket B \rrbracket(p_2) + \lambda_1 \llbracket B \rrbracket(p_1)$

The following fact can be easily proved by inference on the framework of B :

—Claim: If $\langle B, p_1 \rangle \rightarrow \langle B', p'_1 \rangle$ and $\langle B, p_2 \rangle \rightarrow \langle B', p'_2 \rangle$,

then

$$\langle B, \lambda_2 p_2 + \lambda_1 p_1 \rangle \rightarrow \langle B', \lambda_2 p'_2 + \lambda_1 p'_1 \rangle.$$

Then the conclusion is immediately followed.

Next, a presentation of the semantic operation $\llbracket B \rrbracket$ is given on the basis of the framework of the programme B . In order to do this for quantum circles, certain auxiliary mark is needed.

Let J be a quantum programme like $\llbracket J \rrbracket = 0_{H_{all}}$ for all $p \in \Gamma(H)$; say, $J \equiv \text{while } \eta_{\text{trivial}[x]} = 1 \text{ do skip}$, in which

x is a quantum variable, and $\eta_{\text{trivial}} = \{ \eta_0 = 0_{H_x}, \eta_1 = J_{H_x} \}$ is a ordinary measurement on H_x . (while $\eta[\bar{x}] = 1 \text{ do } B$) $0 \equiv$

J , (while $\eta[\bar{x}] = 1 \text{ do } B$) $n+1 \equiv \text{measure } \eta[\bar{x}] : \bar{B}$, in which $\bar{B} \equiv B_0, B_1$, and $B_0 \equiv \text{skip}, B_1 \equiv B$; (while $\eta[\bar{x}] = 1 \text{ do } B$) n for all $n \geq 0$ is fixed.

PROPOSITION 5.1.

$\llbracket \text{skip} \rrbracket(p) = p$. If $\text{type}(x) = \text{Boolean}$, then

$$\llbracket x := 0 \rrbracket(p) = |0\rangle_x \langle 1|_x p |1\rangle_x \langle 0|_x + |0\rangle_x \langle 0|_x p |0\rangle_x \langle 0|_x;$$

and If $\text{type}(x) = \text{integer}$, then

$$\llbracket x := 0 \rrbracket(p) = \sum_{n=-\infty}^{\infty} |0\rangle_x \langle n|_x p |n\rangle_x \langle 0|_x \quad (3)$$

$$\llbracket \bar{x} := \delta \bar{x} \rrbracket(p) = \delta p \delta^+ \quad (4)$$

$$\llbracket B_1 = B_2 \rrbracket(p) = \llbracket B_2 \rrbracket(\llbracket B_1 \rrbracket(p)) \quad (5)$$

$$\left[\text{measure } \eta[\bar{x}] : \bar{B} \right](p) = \sum_m \llbracket B_m \rrbracket(\eta_m p \eta_m^+) \quad (6)$$

$$\llbracket \text{while } \eta[\bar{x}] = 1 \text{ do } B \rrbracket(p) = \bigvee_{n=0}^{\infty} \llbracket \text{while } \eta[\bar{x}] = 1 \text{ do } B \rrbracket^n(p),$$

In which \bigvee represents the least over limit of part denseness

performer on the basis of the below part sequence $\subseteq \alpha_{\text{ROOF}}$.

(1),(2) and (3) are apparent. (4)By Lemma 5.1 and the transition law for sequential article, it is obtained that

$$\begin{aligned} \llbracket B_2 \rrbracket(\llbracket B_1 \rrbracket(p)) &= \llbracket B_2 \rrbracket\left(\sum \left\{ p_1 : \langle B_1, p \rangle \rightarrow^* \langle E, p_1 \rangle \right\}\right) \\ &= \sum \left\{ \llbracket B_2 \rrbracket p_1 : \langle B_1, p \rangle \rightarrow^* \langle E, p_1 \rangle \right\} \\ &= \sum \left\{ \sum \left\{ p' : \langle B_2, p_1 \rangle \rightarrow^* \langle B, p' \rangle \right\} : \langle B_1, p' \rangle \rightarrow^* \langle E, p_1 \rangle \right\} \\ &= \sum \left\{ p' : \langle B_1, p \rangle \rightarrow^* \langle E, p_1 \rangle \text{ and } \langle B_2, p_1 \rangle \rightarrow^* \langle E, p' \rangle \right\} \\ &= \sum \left\{ p' : \langle B_1; B_2, p \rangle \rightarrow^* \langle E, p' \rangle \right\} \\ &= \llbracket B_1 ; B_2 \rrbracket(p). \end{aligned}$$

follows instantly from the transition law for measuring. Two auxiliary performers are brought into:

$$f_i : \Gamma^-(H_{all}) \rightarrow \Gamma^-(H_{all})$$

$$f_i(p) = \eta_i p \eta_i^+$$

for all $p \in \Gamma^-(H)$ and $i = 0, 1$. For simple presentation, while

for expression “while $\eta[\bar{x}] = 1 \text{ do } B$ ” is written. Firstly, it is proved that

$$\llbracket (\text{while})^n \rrbracket(p) = \sum_{k=0}^{n-1} [f_0 \circ (\llbracket B \rrbracket \circ f_1)^k](p)$$

for all $n \geq 1$ by inference upon n . The example of $n = 1$ is apparent. Then, through (1), (4), and (5) which are proved above and this induction assumption on $n - 1$, it is obtained that

$$\begin{aligned} \llbracket (\text{while})^n \rrbracket(p) &= \llbracket B; (\text{while})^n \rrbracket(f_1(p)) + \llbracket \text{skip} \rrbracket(f_0(p)) \\ &= \llbracket (\text{while})^{n-1} \rrbracket(\llbracket B \rrbracket \circ f_1)(p) + f_0(p) \\ &= f_0(p) + \sum_{k=0}^{n-2} [f_0 \circ (\llbracket B \rrbracket \circ f_1)^k](\llbracket B \rrbracket \circ f_0)(p) \quad (5) \\ &= \sum_{k=0}^{n-1} (p) [f_0 \circ (\llbracket B \rrbracket \circ f_1)^k] \end{aligned}$$

Secondly, it follows instantly from Eq. (5) that $\{\llbracket (while)^n \rrbracket(p)\}_{n=0}^{\infty}$ while is a rising order. Thus,

$$\bigvee_{n=0}^{\infty} \llbracket (while)^n \rrbracket(p)$$

exists. Moreover, there is

$$\begin{aligned} \llbracket while \rrbracket(p) &= \sum \{p' : \langle while, p \rangle \rightarrow^* \langle E, p' \rangle\} \\ &= \sum_{n=1}^{\infty} \sum \{p' : \langle while, p \rangle \rightarrow^n \langle E, p' \rangle\} \end{aligned}$$

So it is enough to show that

$$\sum \{p' : \langle while, p \rangle \rightarrow^n \langle E, p' \rangle\} = (p)[f_0 \circ (\llbracket B \rrbracket \circ f_1)]^{n-1}$$

for all $n \geq 1$. This can be done without difficulty by induction upon n .

A recursive portrayal of the semantic operation of a quantum circle can be originated from the precedent proposition.

COROLLARY 5.1. *If it is written while for quantum circle “while $\eta[\bar{x}] = 1$ do B ”, then for any $P \in \Gamma^-(H_{all})$, it thinks*

$$\text{that } \llbracket while \rrbracket(p) = \eta_0 p \eta_0^+ + \llbracket while \rrbracket(\llbracket B \rrbracket(\eta_1 p \eta_1^+))$$

Direct from Proposition 5.1, (6) and Eq. (5).

The proposition listed below demonstrates that a semantic operation does not multiply the trace of denseness performer of quantum variable.

PROPOSITION 5.2. For any quantum program B , it thinks that

$$\xi(\llbracket B \rrbracket(p)) \leq \xi(p) \text{ for all } p \in \Gamma^-(H_{all}).$$

It continues by inference on the framework of B . *Case 1.* $B \equiv \text{skip}$. Apparent *Case 2.* $B \equiv x := 0$. If $\text{type}(x) = \text{integer}$, then

$$\begin{aligned} \xi(\llbracket B \rrbracket(p)) &= \sum_{n=-\infty}^{\infty} \xi(|0\rangle_x \langle n|p|n\rangle_x \langle 0|) \\ &= \sum_{n=-\infty}^{\infty} \xi(|0\rangle_x \langle 0|0\rangle_x \langle n|p|n\rangle_x) \\ &= \xi\left[\left(\sum_{n=-\infty}^{\infty} |n\rangle_x \langle n|\right)p\right] \\ &= \xi(p) \end{aligned}$$

It can be affirmed in a resembling way when $\text{type}(x) = \text{Boolean}$.

Case 3. $B \equiv \bar{x} := \delta \bar{x}$. Then

$$\xi(\llbracket B \rrbracket(p)) = \xi(\delta p \delta^+) = \xi(\delta^+ \delta p) = \xi(p).$$

Case 4. $B \equiv B_1; B_2$. It is followed from the induction

assumption upon B_1 and B_2 that

$$\begin{aligned} \xi(\llbracket B \rrbracket(p)) &= \xi(\llbracket B_2 \rrbracket(\llbracket B_1 \rrbracket(p))) \\ &\leq \xi(\llbracket B_1 \rrbracket(p)) \\ &\leq \xi(p) \end{aligned}$$

Case 5. $B \equiv \text{measure } \eta[\bar{x}] : \bar{B}$. Next, by induction assumption, it is obtained that

$$\begin{aligned} \xi(\llbracket B \rrbracket(p)) &= \sum_m \xi(\llbracket B_m \rrbracket(\eta_m p \eta_m^+)) \\ &\leq \sum_m \xi(\eta_m p \eta_m^+) \\ &= \xi\left[\left(\sum_m \eta_m^+ \eta_m\right)p\right] \\ &= \xi(p) \end{aligned}$$

5. Examples and illustration

Example 6.1. Assume that $\text{type}(x_1) = \text{Boolean}$ and $\text{type}(x_2) = \text{integer}$. Regard the programme:

$$\begin{aligned} B &\equiv x_1 := 0; x_2 := 0; x_1 := H_{x_1}; x_2 := x_2 + 2; \\ &\text{measure } \eta[x_1] : \bar{S} \end{aligned}$$

In which:

— η is the measurement on the basis the computational base $\{|0\rangle, |1\rangle\}$ of H_2 ; that is, $\eta = \{\eta_0, \eta_1\}$, $\eta_0 = |0\rangle\langle 0|$ and $\eta_1 = |1\rangle\langle 1|$;

$$\text{— } \bar{B} \equiv B_1, B_2, \text{ and}$$

$B_1 \equiv \text{skip}$;

$B_2 \equiv \text{while } N[x_2] = 1 \text{ do } x_1 := y_z x_1$, in which y_z is a Pauli matrix,

$$N = \{N_0, N_1\}, N_0 = \sum_{n=-\infty}^0 |n\rangle\langle n| \text{ and } N_1 = \sum_{n=1}^{\infty} |n\rangle\langle n|.$$

Let

$$p_0 = \bigotimes_{x \neq x_1, x_2} |0\rangle_x \langle 0|$$

And

$$p = |1\rangle_{x_1} \langle 1| \otimes | -1\rangle_{x_2} \langle -1| \otimes p_0.$$

Then these computations of B beginning in p are

$$\begin{aligned}
 &\langle B, p \rangle \rightarrow \langle x_2 := 0; x_1 := H_{x_1}; x_2 := x_2 + 2; \text{measure}, p_1 \rangle \\
 &\rightarrow \langle x_1 := H_{x_1}; x_2 := x_2 + 2; \text{measure}, p_2 \rangle \\
 &\rightarrow \langle x_2 := x_2 + 2; \text{measure}, p_3 \rangle \\
 &\rightarrow \langle \text{measure}, p_4 \rangle \\
 &\rightarrow \left\{ \begin{array}{l} \langle B_1, p_5 \rangle \rightarrow \langle E, p_5 \rangle \\ \langle B_2, p_6 \rangle, \end{array} \right. \\
 &\quad \langle B_2, p_6 \rangle \rightarrow \langle x_1 := y_z x_1; B_2, p_6 \rangle \\
 &\rightarrow \langle B_2, -p_6 \rangle \\
 &\rightarrow \dots \\
 &\rightarrow {}^{2^{n-1}} \langle x_1 := y_z x_1; B_2, (-1)^{n-1} p_6 \rangle \\
 &\rightarrow \langle B_2, (-1)^n p_6 \rangle \\
 &\rightarrow \dots
 \end{aligned}$$

In which measure represents the expression “measure $\eta[x_1]: \bar{B}$ ”, and

$$\begin{aligned}
 p_1 &= |0\rangle_{x_1} \langle 0| \otimes |-1\rangle_{x_2} \langle -1| \otimes p_0 \\
 p_2 &= |0\rangle_{x_1} \langle 0| \otimes |0\rangle_{x_2} \langle 0| \otimes p_0 \\
 p_3 &= |+\rangle_{x_1} \langle +| \otimes |0\rangle_{x_2} \langle 0| \otimes p_0 \\
 p_4 &= |+\rangle_{x_1} \langle +| \otimes |2\rangle_{x_2} \langle 2| \otimes p_0 \\
 p_5 &= \frac{1}{2} |0\rangle_{x_1} \langle 0| \otimes |2\rangle_{x_2} \langle 2| \otimes p_0 \\
 p_6 &= \frac{1}{2} |1\rangle_{x_1} \langle 1| \otimes |2\rangle_{x_2} \langle 2| \otimes p_0.
 \end{aligned}$$

So B can deviate from p . Notice that B_2 also possesses the transition

$$\langle B_2, (-1)^n p_6 \rangle \rightarrow \langle E, 0_{H_{all}} \rangle$$

but these transitions are all the time abandoned where the part denseness performer of the targeted arrangement is the zero performer.

Example 6.2. Let $\text{type}(x) = \text{integer}$, and let

$$\begin{aligned}
 \eta_0 &= \sum_{n=1}^{\infty} \sqrt{\frac{n-1}{2n}} (|-n\rangle \langle -n| + |n\rangle \langle n|), \\
 \eta_1 &= \sum_{n=1}^{\infty} |0\rangle \langle 0| + \sqrt{\frac{n+1}{2n}} (|-n\rangle \langle -n| + |n\rangle \langle n|),
 \end{aligned}$$

Then $\eta = \{ \eta_0, \eta_1 \}$ is a yes-no measuring on the situation space H_x of quantum variable x . Think the quantum circle:

while $\eta[x] = 1$ do $x := x + 1$.

For simple presentation, it is written that when for this programme. Let $p_0 = \bigotimes_{x' \neq x} |0\rangle_{x'} \langle 0|$

and $P = |0\rangle_x \langle 0| \otimes p_0$. Then

$$\llbracket (\text{while})^n \rrbracket (p) = \begin{cases} 0_{H_{all}} & \text{if } n=0,1,2, \\ \frac{1}{2} \left(\sum_{k=0}^{n-1} \frac{k-1}{k!} |k\rangle_x \langle k| \right) \otimes p_0 & \text{if } n \geq 3, \end{cases}$$

$$\llbracket \text{while} \rrbracket (p) = \frac{1}{2} \left(\sum_{n=2}^{\infty} \frac{n-1}{n!} |n\rangle_x \langle n| \right) \otimes p_0$$

$$\xi(\llbracket \text{while} \rrbracket (p)) = \frac{1}{2} \sum_{n=2}^{\infty} \frac{n-1}{n!} = \frac{1}{2}.$$

This denotes that programme while ends upon input p with possibility $\frac{1}{2}$, and it deviates from input p with possibility $\frac{1}{2}$.

To make a conclusion of this part, it is seen that how quantum programmes transform the situations of quantum variable and how they gain access to quantum variable.

Let $\mu \subseteq \text{Var}$ be a series of quantum variable. For any performer $\gamma \in L(H_{all})$, it is written that

$$\xi_{\mu}(\gamma) \text{ for the part trace } \xi_{\otimes_{x \in \mu} H_x}(\gamma).$$

6. Conclusions

In the article, operational system, rule setting and semantics environment of virtual laboratory have been studied. Furthermore, the initiation of semantics environment and the input and output of virtual laboratory have been clear and definite. And virtual quantum laboratory have been confirmed. When it comes to put into use, the single performers and measurements in quantum programmes are often selected from a specific fix, which is probably general; say, if a programme merely has quantum variable of form Boolean, then the single performers can be got from the normal series of general gates, comprising the Hadamard, phase, CNOT, and $\pi/8$ gates and the measuring in the computational base is required merely. For simple presentation, nevertheless, these single performers and measurements are not selected to specify since selecting them does have any vital distinction in the theory researched in this composition.

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8. References

1. Oke Peter R, Proctor Roger, Rosebrock Uwe, *et al.* The Marine Virtual Laboratory (version 2.1): enabling efficient ocean model configuration [J], *Geoscientific model development*, 2016; 9(9):3297-3307.
2. Al-Khalifa Hend S. CHEMOTION. A gesture based chemistry virtual laboratory with leap motion [J], *computer applications in engineering education*. 2017; 25(6):961-976.
3. Dehn Frank. Concrete - Is it a Construction Material from the virtual Laboratory? [J], *beton- und stahlbetonbau*. 2017; 122(11):703-703.
4. Ortega-Moody, Jorge-Alberto; Sanchez-Alonso, Roger-Ernesto; Grise, William R; *et al.* Virtual laboratory of industrial scenarios for training in the areas of automation and control [J], *Dyna*, 2017; 92(3):285-287.
5. Popovic Natasa, Naumovic Milica B. Virtual laboratory and learning management system in optimal control theory education [J], *international journal of electrical engineering education*. 2016; 53(4):357-370.
6. Jezierska Karolina, Podraza Wojciech, Domek Hanna, *et al.* The Virtual Laboratory for Student Understanding of Biophysics [J]. *National academy science letters-India*. 2016; 39(4):295-299.
7. Gomez-Sacristan Angel, Sempere-Paya Victor M, Rodriguez-Hernandez, Miguel A. Virtual Laboratory for QoS Study in Next-Generation Networks With Metro Ethernet Access [J], *Ieee transactions on education*. 2016; 59(3):187-193.
8. Liu Wen. The design and implementation of intelligent computer aided instruction system based on WEB [J], *agro food industry hi-tech*. 2017; 28(3):1342-1344.
9. Xiao Hongyun. Research on the Construction of Computer Aided Instruction System Based on Comprehensive Education Model [J], *agro food industry hi-tech*. 2017; 28(1):1167-1171.
10. Siqueira, Ana Rita; Nogueira, Maria Alice de Lima Gomes. Focalizando um segmento específico da rede privada de ensino: escolas particulares de baixo custo Focusing on a specific segment of the private education system: low-cost private schools [J], *Educação e Pesquisa*. 2017; 43(4):1005-1022.
11. Portillo-Torres, Mauricio Cristhian. Educación por habilidades: Perspectivas y retos para el sistema educativo Skills education: prospects and challenges for the education system [J], *Revista Educación*. 2017; 41(2):118-130.
12. Sergeev A. onward to international free education system in dermatology [j], *international journal of dermatology*. 2017; 56(11):1275-1276.
13. Kobayashi Toru, Arai Kenichi, Sato Hiroyuki, *et al.* An Application Framework for Smart Education System Based on Mobile and Cloud Systems [J], *Ieice transactions on information and systems*. 2017; e100d(10):2399-2410.
14. RyojiMatsuoka. Inequality of effort in an egalitarian education system [J], *Asia Pacific Education Review*. 2017; 18(3):347-359.
15. Abramo Giovanni, D'Angelo Ciriaco Andrea, Rosati Francesco. The north-south divide in the Italian higher education system [J], *Scientometrics*. 2016; 109(3):2093-2117.
16. Lopez Lluch, David Bernardo; Lorenzo Zapata, Javier; del Campo Gomis, Francisco Jose. Preliminary analysis of farm workers perception about continuous education system in the Region of Murcia [J], *revista de la facultad de ciencias agrarias*. 2016; 48(2):177-193.
17. Dutta Angshu, Dutta Himangshu. Some drawbacks of the higher education system in India [J], *current science*. 2015; 109(12):2174-2175.
18. Gong Xiaoli, Liu Yanjun, Jiao Yang, *et al.* A Novel Earthquake Education System Based on Virtual Reality [J], *Ieice transactions on information and systems*. 2015; E98D(12):2242-2249.
19. Zhang Yulai, Zhang Hongxing, Zheng Qingchuan. What regulates the catalytic activities in AGE catalysis? An answer from quantum mechanics/molecular mechanics simulations [J], *physical chemistry chemical pH*. 2017; 19(47):31731-31746.
20. Alhaidari AD. Reconstructing the Potential Function in a Formulation of Quantum Mechanics Based on Orthogonal Polynomials [J], *communications in theoretical physics*. 2017; 68(6):711-728.
21. Aerts Diederik, de Bianchi, Massimiliano Sassoli; Sozzo, Sandro. The Extended Bloch Representation of Entanglement and Measurement in Quantum Mechanics [J], *International journal of theoretical physics*. 2017; 56(12):3727-3739.
22. Fernandez Francisco M, Garcia Javier, Semoradova Iveta, *et al.* Ad Hoc Physical Hilbert Spaces in Quantum Mechanics [J], *International journal of theoretical physics*. 2015; 54(12):4187-4203.