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Mathematical models for the determination of the uplift resistance of a gas pipeline

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Abstract
Transmission or transportation of gas is usually through piping network. The pipings are close conduits of circular crossection. The gas might be transported above or below the earth surface or below the sea bed. The pipes are required to withstand the internal fluid pressure. Gas transmission lines operates at very high pressures. Subject to the type of material the pipe is made of, there is required optimal pipe thickness to withstand the internal fluid pressure. Pipelines do run through places of diverse temperature gradients and the flowing fluid steam is at a bulk temperature. A laid pipeline is subject to different loads: the weight of the pipe, the weight of gas, the weight of the coating materials, the weight of the overlying water or earth mass for buried pipeline. Under the loading conditions the pipes witness different measures of stresses such as temperature stress, longitudinal stress, circumferential stress and radial stress. To provide the right type of supports for a laid pipeline with the view of avoiding buckling of the pipe in service the different loading conditions and the stresses developed must be properly considered. The scope of this work is to generate mathematical models to enable the determination of the optimal wall thickness for the pipe subject to the different loading conditions and the restoring moment to avoid buckling of the pipe for a laid pipeline.

Keywords: uplift resistance, earth mass centroid, maximum deflection, internal fluid pressure, support reaction, buckling load

1. Introduction

Gas pipelines assets and facilities are expensive capital intensive production items designed to have a long service life. Failure of this vital production assets might have serious economic and environmental implications. There published works on uplift mechanisms of buried pipeline caused by creep, earth movement, inadvertently resulting in infilling of soil at the base of the pipe [1, 2, 3, 4, 5]. This situation will result in pressure build up at the base of the pipe, leading to gradual drift of the pipe to the earth surface. The worst case situation is the buckling of the pipe.

Oil and gas pipeline mechanical design concepts based on ANSI/ASME set of standard codes adequately specified design equations for pipe wall thickness, flow velocity, compressors, valves, fittings and flanges design, pipe support spacing, flow density and pipe internal diameter. The applicable set of standard codes are: ANSI/ASME B31, 3, B31.4, B31.8 and API RPI4C [6, 7, 8, 9].

The easiest, safest and the most efficient and economical means of transportation or transmission of fluid be it liquid or gas is through interconnected pipes generally referred to as pipeline network system. In gas transmission system, the pipeline is in three subcategory namely: the gathering pipeline (field pipeline), the main trunk line and the service or distribution line. By virtue of the terrain transversed by the pipeline, weight of the pipes and that of the fluid being conducted, the pipeline might be subject to diverse stresses and subsequently buckling [10]. This necessitates the need for model formulation to determine the safe pipe wall thickness, the uplift resistance of the pipe that will prompt accurate pipe support spacing on the basis of deflection of the pipe.

2. Purposes and Significance

Uplift mechanisms of a buried pipe or the prevailing conditions militating for the upward drift of a buried pipe had been reviewed. Oil and gas pipelines mechanical design concepts were reviewed in all its details. This work strongly rooted on the formulation of mathematical models for the uplift resistance of laid buried pipeline, is to enable the determination of optimal pipe wall thickness, support reactions and pipe deflection. The pipeline irrespective of the length is imagined as supported at the ends. It is believed if the pipe deflection is known to a measure of accuracy pipe supports can be installed along the pipeline to annul the deflection to avoid catastrophic failure due to buckling.

3. Model formulations

The pipe is assumed cylindrical in shape. The internal diameter of the pipe is d_1 and the pipe length is L. The internal fluid pressure is P (Figure 1). The first step in the model formulation is to determine the nominal pipe thickness to withstand the fluid pressure. Subject to the internal fluid pressure, three types of stresses develop, Viz: hoop or circumferential stress, longitudinal stress and radial stress.

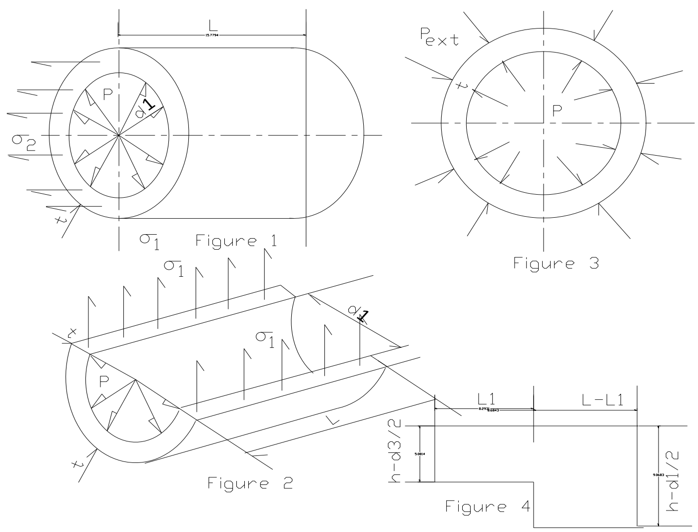
4. Models for stress determination

The expression for the hoop stress is developed assuming the pipe is cut in halves through the diametral plane (Figure 2). Balancing the fluid pressure force and the force acting on the wall of the cylinder circumferentially.

$$2\sigma_1 Lt = Pd_1 L$$
$$\sigma_1 = \frac{Pd_1}{2t} \qquad (1)$$

Where
P-average stream pressure (N/m^2)
σ_1 --hoop stress or circumferential stress (N/m^2)
L-pipe length (m)
t, $\bar{t}$ -pipe thickness (m)
d_1-pipe internal diameter (m)

The longitudinal stress acts along the pipe wall parallel to the longitudinal axis (Figure 1). Balancing forces equally;



$$\sigma_2 \times \pi d_1 t = P \times \pi d_1^2 / 4$$
$$\sigma_2 = \frac{Pd_1}{4t} \qquad (2)$$

Where
σ_2 --longitudinal stress (N/m^2)

The radial stress is obtained by balancing forces along the mean diameter of the pipe. The pipe mean diameter is expressed as;

$$d_m = \frac{d_1 + d_1 + t}{2} = d_1 + t/2 \qquad (3)$$

d_m—pipe mean diameter (m)
The expression for the radial stress goes thus;

$$\pi\sigma_3(d_1 + t/2)L + P_{ext}\pi(d_1 + t)L = P\pi d_1 L$$
$$\therefore \quad \sigma_3 = \frac{Pd_1 - P_{ext}(d_1 + t)}{d_1 + t/2} \qquad (4)$$

Where
P_{ext}—resultant external loads on the pipe (N/m^2)
σ_3 --radial stress (N/m^2)

The resultant external loads on the structure is expressed as:

$$P_{ext} = P_{atm} + P_{earth} + P_c + P_w$$
$$P_{atm} = 1bar$$
$$P_{earth} = \rho_s gh$$
$$P_W = \varphi P_g \quad at \quad (33^\circ C)$$

Where

P_{ext}-resultant external loads on the pipe (N/m^2)
P_{atm}-atmospheric pressure (N/m^2)
P_{earth}-pressure due to overlying earth (N/m^2)
P_{c}-pressure due to the weight of the concrete coating (N/m^2)
P_{w}-pressure due to atmospheric water vapor water (N/m^2)

If the pipe is encased in a concrete of thickness t_1, the load intensity is expressed as,

$$P_c = \frac{F_c}{A_c} = \frac{m_c g}{A_c} = \frac{\rho_c V_c g}{A_c}$$
$$= \frac{\rho_c t_1 g(d_2 + d_1)}{2d_1}$$
$$A_c = \pi d_2 L$$
$$V_c = \frac{\pi(d_2^2 - d_1^2)L}{4} = \frac{\pi(d_2 + d_1)(d_2 - d_1)L}{4}$$
$$= \pi t L(d_2 - d_1)$$
$$d_2 - d_1 = 2t$$

Where

Fc-Force exerted by the casibg (N)
m_c-mass of the concrete casing (kg)
t_1-casing thickness (m)
ρ_c-density of casing material (kg/m3)
V_c-volume of concrete (m^3)
A_c-inner curved surface area of the concrete coating (m^2)
d_2-pipe outer diameter (m)
V_c-volume of concrete (m^3)
V_p-pipe volume (m^3)
V_e-earth volume (m^3)
V_g-volume of gas in the pipeline (m^3)

Subject to the tri-axial stress condition, the maximum shear stress is the greatest of the three values.

$$\tau_{1\max} = \left|\frac{\sigma_1 - \sigma_2}{2}\right| \; or \; \left|\frac{\sigma_2 - \sigma_3}{2}\right| \; or \; \left|\frac{\sigma_3 - \sigma_1}{2}\right|$$
$$\therefore \quad \tau_{1\max} = \left|\frac{\sigma_3 - \sigma_1}{2}\right| \qquad (5)$$

Where

σ_3 --radial stress (N/m^2)
$\tau_{1\max}$--maximum shear stress under tri-axial stress condition (N/m^2)
$\tau_{\max}$--induced maximum shear stress in the pipe (N/m^2)

Under uni-axial stress condition, $\sigma_2,\ \sigma_3 = 0$

$$\tau_{2\max} = \frac{\sigma_1}{2} \tag{6}$$

$\tau_{2\max}$ --maximum shear stress under uni-axial stress condition (N/m^2)

Temperature stress in the system is expressed as;

$$\begin{aligned}\sigma_T &= E\varepsilon = E\frac{\Delta L}{L}\\ &= E\frac{\alpha L\Delta T}{L}\\ &= E\alpha\Delta T\end{aligned} \tag{7}$$

Where

σ_T --temperature stress (N/m^2)
ε --poisson ratio
E-Young's modulus of elasticity for the pipe (N/m^2)
A-linear expansivity of the pipe material (/°C)
ΔT-temperature difference between the pipe gas and the environment (°C)
ΔL-change in length of the pipe (m)

On the basis of these analyses the overall induced maximum shear stress in the pipe can be expressed as:

$$\begin{aligned}\tau_{\max} &= \tau_{1\max} - \tau_{2\max}\\ &= \frac{\sigma_3 - \sigma_1}{2} - \frac{\sigma_T}{2}\end{aligned} \tag{8}$$

Applying failure (yielding) analysis known as maximum shear stress theory. The theory is based on the assumption that the pipe will fail or yield when the maximum induced shear stress reaches a value equal to the sear stress at the instant of failure or yielding in a simple tension test. Hence the allowable shear stress in the pipe is given as,

$$\tau_a = \frac{0.5\sigma_{ypt}}{FS} = \frac{\sigma_{ypt}}{2\,FS} \tag{9}$$

$$\therefore\ \frac{|\sigma_3 - \sigma_1|}{2} - \frac{\sigma_T}{2} = \frac{\sigma_{ypt}}{2\,FS}$$

$$\sigma_1 - \sigma_3 - \sigma_T = \frac{\sigma_{ypt}}{FS} \tag{10}$$

σ_{ypt} is the yield point of the pipe material under uni-axial tension test and FS is the factor of safety.
Substituting equations 1, 4 and 7 in equation 10

$$\frac{Pd_1}{2t} - \frac{Pd_1 - P_{ext}(d_1 + t)}{d_1 + t/2} - \sigma_T = \frac{\sigma_{ypt}}{FS}$$

$$t^2\left[-2P_{ext} - \sigma_T - \frac{\sigma_{ypt}}{FS}\right] + t\left[-1.5Pd_1 + 2P_{ext}d_1 - 2\sigma_T d_1 - \frac{2\sigma_{ypt}}{FS}\right] + pd_1 = 0 \tag{11}$$

$$a = -2P_{ext} - \sigma_E - \frac{\sigma_{ypt}}{FS}$$

$$b = -1.5Pd_1 + 2P_{ext}d_1 - 2\sigma_T d_1 - \frac{\sigma_{ypt}}{FS}$$

$$c = Pd_1$$

$$\bar{t} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(12)$$

$\bar{t}$ is the design wall thickness of the pipe for safe operation with due regard to the required operating conditions.

4.1 Determination of the Buckling Resistance of the Pipe

Consider a pipe of length L, internal diameter d_1, thickness t and external diameter d_2. The internal fluid pressure is P. It is coated with concrete over a length L_1 of its length and buried to a depth h inside the ground. It is mounted on two supports as shown in fig. 5.

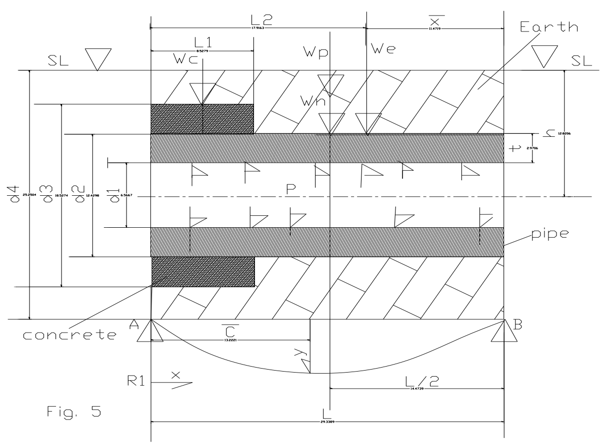


Fig. 5

The centroid of the earth mass above the pipe is expressed as:

$$A_e\bar{x} = A_{e1}x_1 + A_{e2}x_2$$

$$A_{e1} = (L - L_1)\left(h - \frac{d_2}{2}\right), \quad A_{e2} = L_1\left(h - \frac{d_3}{2}\right)$$

$$x_1 = \frac{L - L_1}{2}, \quad x_2 = \frac{2L - L_1}{2}$$

$$\bar{x} = \frac{A_{e1}x_1 + A_{e2}x_2}{A_e}$$

$$\bar{x} = \frac{(L - L_1)\left(h - \frac{d_2}{2}\right)\left(\frac{L - L_1}{2}\right) + L_1\left(h - \frac{d_3}{2}\right)\frac{2L - L_1}{2}}{(L - L_1)\left(h - \frac{d_2}{2}\right) + L_1\left(h - \frac{d_3}{2}\right)} \qquad (13)$$

Load Components

(i) Pipe Weight

$$W_p = m_p g = \rho_p V_p g = \frac{\pi L_3 \rho_p g (d_1^2 - d_2^2)}{4} \qquad (14)$$

(ii) Concrete Weight

$$W_c = m_c g = \rho_c V_c g = \frac{\pi L_1 \rho_c g (d_3^2 - d_2^2)}{4} \quad (15)$$

(iii) Earth Mass Weight

$$\begin{aligned} W_e &= m_e g = \rho_e V_e g \\ &= \pi \rho_e g \left[h^2 L - \frac{d_2^2 L}{4} - \frac{(d_3^2 - d_2^2) L_1}{4} \right] \\ V_e &= \pi \left[h^2 L - \frac{d_2^2 L}{4} - \frac{(d_3^2 - d_2^2) L_1}{4} \right] \end{aligned} \quad (16)$$

Weight of gas inside the pipe

$$\begin{aligned} m_n &= \frac{PV_g}{ZRT} \\ R &= \sum \frac{m_i}{m_H} R_i, \quad m_i = n_i M_i \\ W_n &= m_n g \end{aligned} \quad (17)$$

Where
m_p-mass of pipe (Kg)
m_c-mass of concrete (Kg)
m_e-mass of overlying earth (Kg)
m_H-mass of hydrocarbon constituents (Kg)
m_i-mass fraction of the constituents (Kg)
n_i-number of moles of the constituents
M_i-molar mass of the constituents (Kg)
R-average gas constant for the constituents (J/KgK)
R_i-gas constant for the constituents (J.KgK)
g-gravitational acceleration (m/s^2)
W_p-weight of pipe (N)
W_c-weight of concrete (N)
W_e-weight of overlying earth mass (N)
W_g-weight of pipe (N)
F-restoring force (N)
R_1, R_2-reactions at the supports (N)

(iv) The reaction at the supports are R_1 and R_2.
Consider all the loads as point loads.
Applying Macaulay's method [10], for the moment distribution along the pipeline, treating all the loads as point loads. Moment distribution is referenced to point A. Anti-clockwise moment is taken as positive.

$$\begin{aligned} EI \frac{d^2 y}{dx^2} &= M \\ &= R_1 (W_p + W_n) x - W_c (x - L_1) - W_p (x - L/2) - W_n (x - L/2) - W_e (x - L_2) \\ &= R_1 x - W_c (x - L_1) - (W_p + W_n)(x - L/2) - W_e (x - L_2) \end{aligned} \quad (18)$$

$$EI\,\frac{dy}{dx} = \frac{R_1x^2}{2} - W_c\left(\frac{x^2}{2} - L_1x\right) - \left(W_p + W_n\right)\left(\frac{x^2}{2} - \frac{L}{2}\right) - W_e\left(\frac{x^2}{2} - L_2x\right) + A_1 \quad (19)$$

$$EIy = \frac{R_1x^3}{6} - W_c\left(\frac{x^3}{6} - \frac{L_1x^2}{2}\right) - \left(W_p + W_n\right)\left(\frac{x^3}{6} - \frac{Lx^2}{4}\right) - W_e\left(\frac{x^3}{6} - \frac{L_2x^2}{2}\right) + A_1x + B_1 \quad (20)$$

$$Applying\ \ the\ boundary\ \ conditions\ \ when\ \ x = 0, y = 0\ and\ \ when\ \ x = L,\ y = 0$$

$$B_1 = 0$$

$$A_1 = \frac{L}{2}\left[R_1 - W_c + \frac{W_p + W_c}{2} - W_e\right] + \frac{L}{2}\left[L_1W_e + L_2W_e\right]$$

To determine the reaction at support A.

$$R_1 + R_2 = W_C + W_e + W_p + W_n$$

Taking moment about point B

$$R_1L = W_C\left(\frac{2L - L}{2}\right) + \left(W_P + W_C\right)L/2 + W_e\bar{x}$$

$$R_1 = \frac{W_C\left(\frac{2L - L}{2}\right) + \left(W_P + W_C\right)L/2 + W_e\bar{x}}{L}$$

$$R_2 = \left(W_C + W_e + W_P + W_C\right) - R_1$$

At the point of maximum deflection, dy/dx=0, hence,

$$0 = \frac{R_1x^2}{2} - W_c\left(\frac{x^2}{2} - L_1x\right) - \left(W_P + W_n\right)\left(\frac{x^2}{2} - \frac{Lx}{2}\right) - W_e\left(\frac{x^2}{2} - L_2x\right) + A_1$$

$$x^2\left[\frac{R_1}{2} - \frac{W_C}{2} - \left(\frac{W_P + W_n}{2}\right) - \frac{W_e}{2}\right] - x\left[W_cL_1 + \frac{(W_P + W_n)L}{2} + W_eL_2\right] + A_1 = 0$$

$$a_1 = \frac{R_1}{2} - \frac{W_C}{2} - \left(\frac{W_P + W_n}{2}\right) - \frac{W_e}{2}$$

$$b_1 = W_cL_1 + \frac{(W_P + W_n)L}{2} + W_eL_2$$

$$c_1 = A_1$$

$$\bar{c} = \frac{-b_1 \pm \sqrt{b_1^2 - 4a_1c_1}}{2a_1} \quad (21)$$

Substituting equation 20 in 19, to obstain the maximum deflection.

$$y = \left[\frac{R_1\bar{C}^3}{6} - W_C\left(\frac{\bar{C}^2}{6} - \frac{L_1\bar{C}^2}{2}\right) - \left(W_p + W_n\right)\left(\frac{\bar{C}^2}{6} - \frac{L\bar{C}^2}{4}\right) - W_e\left(\frac{\bar{C}^2}{6} - \frac{L_2\bar{C}^2}{2}\right) + A_1\bar{C}\right] / EI$$

The bending moment at the point of maximum deflection is given by the expression;

$$M = R_1\overline{C} - W_C\left(\overline{C} - L_1\right) - \left(W_P + W_n\right)\left(\overline{C} - L/2\right) - W_e\left(\overline{C} - L_2\right)$$

The restoring force, F, at point if maximum deflection is gives as

$$Fy = M, \qquad F = M / y$$

This is the required force to prevent buckling of the pipeline subject to the loading conditions. This force can be offered by providing additional support at point $\overline{C}$ or overdesigning the pipe to be of such thickness to withstand the buckling load F. The buckling load F is referred as the uplift resistance of the pipe.

5. Model Applications

The concept of uplift resistance in this work could be applied to oil or gas pipelines network system to provide additional supports to prevent buckling of the pipeline. It is a well known fact that oil and gas pipelines are laid over a distance of thousands of kilometer and they are expensive production assets that last for long period of time. Hence the structure should be well designed with adequate support spacing.

6. Recommendations

Computer simulated programming algorithm should be incorporated to enable the determination of the buckling load, pipe support spacing and other relevant parameters for more efficient design, construction and installation of oil and gas pipelines network system.

7. Conclusions

Mathematical models have been developed for the determination of safe pipe wall thickness, uplift resistance, and the point of maximum pipe deflection to enable proper spacing of pipe support. This is to avoid failure of the pipe by the reason of fatigue and bending tresses that will ultimately result in the buckling of the pipe under service conditions. Reviewed works of focused on uplift mechanisms of buried pipes but not on pipelines built on supports.

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