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A review paper on automatic design of sheet metal forming processes by 'un-forming'

Md Zeeshan[1], Pushpendra Kushwah[2], Vinay Kumar[3]
[1-3] Department of Mechanical Engineering, Indraprastha University, MVSIT, Sonipat, Mahavir Swami Institute of Technology, Sonipat Haryana, India

Abstract
Most sheet metal components are made by deep drawing, which requires expensive tooling. Although many new flexible forming processes have been invented, they have largely not had industrial application, so it would be valuable if intelligent means to design new processes existed. This has not previously been attempted, although there has been work to classify both products and processes and to define optimal forming processes. A body of work in garment production examines the optimal flattening of garments, starting from their final form on a human body, to deduce the best cutting pattern
from flat fabric. This paper develops a related approach for the first time, "un-forming" sheet metal from its finished geometry to a flat blank without prior specification of a process. An algorithm is developed that allows specification of process constraints and great freedom in implementing un-forming strategies, leading to a prediction of the strain history of the un-forming process. Reversing the direction of this history, allows prediction of the stresses in the workpiece required to form the target part, by use of an appropriate material model. The external forces (boundary conditions) required to maintain equilibrium with this stress state can then be calculated, allowing an iterative refinement of the constraints on unforming until a physically achievable process has been designed. The approach is how several previously untried forming strategies could lead to specification of new process designs. In future work, the method could be extended to allow an iterative specification of tooling to create the required boundary conditions, and hence to complete automatic process designs.

Keywords: complete, components, expensive, specification

Introduction

Sheet metal components are ubiquitous, but options to produce them are relatively limited. Existing mass production processes such as deep drawing are inflexible, and consequently expensive for small batch, high-value production. In addition, it takes time to produce and set up the required tooling. Furthermore, given the mechanics of these processes, half of all sheet metal never makes it into a product, having been cut off during manufacturing [1]. There has been great interest in the invention and exploration of flexible processes for the past 30 years, aiming automate the skills of the craftsmen of earlier times. However, this has proved challenging: automated design of toolpaths for spinning remains elusive [2]; incremental sheet forming is limited by poor accuracy and high residual stress [3]; other novel processes have not flourished outside laboratory conditions. Thorough reviews of flexible forming processes are given by Jeswiet *et al*. [4], Allwood and Utsunomiya [5] and Groche [6]. Innovations in the technology of flexible metal forming have to date arisen either from the product of laborious experimental study or out of isolated eureka moments. Is it possible to design?
processes automatically? Surveys in the literature can be used to map available processes and identify gaps. Similarly, the literature can inform some design choices, but it does not provide means to design future machines for particular forming requirements. This paper therefore asks whether, rather than relying on accidents of
innovation, structured means could be developed to design new sheet metal forming processes to meet customers' needs.

Literature Review

Automatic design of new processes has not previously been attempted, but several strands of literature provide a foundation for work in this area. These include work on the classification of processes and products, the design of cutting patterns for garments, design of optimised blanks in deep drawing and the specification of criteria for optimal forming. These are briefly reviewed in turn.
The German DIN standard [7] presents a widely used classification of known processes to facilitate selection. Allwood [8] built on this aiming to envisage every possible configuration of tooling around a roll bite in ring rolling, inspired by Zwicky's idea [9] of the morphological box. This was extended in a subsequent paper [10] to create a structured map of all possible configurations of any tools around any workpiece. However, although this appears exhaustive, the approach gives no insight into process mechanics and hence the feasibility of the different configurations. Several authors have classified part geometry with the intention of automating the selection of process routes for differenparts. Lie *et al*. [11] present an automatic feature extraction system for prismatic raw materials. Nasr and Kamrani [12] also attempt to automate feature recognition in an effort to bridge design and manufacturing. Similarly, Li *et al*. [13] focuses on parts from the aviation industry, but extends existing feature recognition to consider a holistic attribute adjacency graph (HAAG). In future work part classification of this type could be used to allow the design of a forming process to create a family of parts, but the work in this paper will be restricted to creating a single target geometry.

Modelling the manufacturing process in reverse has created widespread interest in academic literature related to the garment industry, where complex final surfaces are defined by the body shape for which the garment is designed. Starting from this final target shape, the challenge for garment manufacturing is to
design the cutting patterns leading to minimum waste of material and easy garment assembly. Several attempts have been made to develop algorithms which "un-form" the final garment into several flat components, using the locations of highest strain to optimise the pattern of cuts in the initial sheets of fabric. Early examples of these approaches are reviewed by McCarteney *et al*. [14], and more recently in Wang *et al*. [15]. A simplified mass-spring system is commonly used to model the 3D surface, with a subsequent optimisation that minimises the elastic energy, distances between nodes or the change in local area. This approach is similar to the challenge addressed in this paper, but differs in two key respects:the model of fabric deformation assumes elastic behaviour, where sheet forming must be plastic; the final garment is assembled from several components, where the ambition of this paper is to design the process for forming a single component. A body of work related to this problem in the garment industry, the parameterisation of 3D surfaces to a 2D plane is a distinct field of research in applied mathematics with applications in cartography and computer graphics among others. Although this multi-disciplinary interest in parameterisation has led to a large number of
computational tools, most of them are not concerned with material bodies, hence do not produce physically realistic "flattenings" as shown by Hinds [16]. Azariadis and Aspragathos [17] and later Azariadis *et al*. [18], present some of the early ideas in computational parameterisation for 3D surfaces. They discretised the surface and flattened individual elements, allowing gaps to form between them, while later bridging those gaps using a geometric objective. However, their approach has no material model,aiming instead to minimise an overall metric of element distortion, so this cannot be translated directly to metal forming.Within the literature of metal forming, inverse Finite Elements Methods have been used for various processes, but mainly for blank design in deep drawing: given a target product geometry,and hence tool shape, what shape should be cut from flat sheet (the blank) prior to forming? Lee and Huh [19] use an inverse method to produce blank shapes by minimising the plastic deformation energy, based on Hencky's deformation theory and Hill's anisotropic yield criterion. Cai *et al*. [20] used deformation non-uniformity as an objective function, along with volume constancy, aiming to minimise material waste. The initialisation of thesimulation happens by moving all the nodes towards the target plane, which can cause numerical difficulties when analysing complex shapes. This approach relates to the ambition of this paper, but can only be used for a pre-defined process – which thus constrains the boundary conditions on the deformation of the workpiece.

One final area of literature relevant to the ambition of process design is the work by Chung and Richmond [21, 22] aiming to specify the characteristics of optimal forming processes, defined as those which minimise the work of deformation across the workpiece. Under broad assumptions, they demonstrate that the deformation path of minimum work is one which has minimum effective strain across the workpiece. A practical implication of this insight is that by minimising the plastic work of deformation, the likelihood of failure for a given part geometry is reduced. However, to date, this approach has not been used for process design. This review has demonstrated several strands of work that provide building blocks of knowledge towards an algorithm for designing sheet metal forming processes. Prior work on process classification has demonstrated the scope for innovation, but not addressed the physical constraints of deformation. Work on garment flattening and surface parameterisation was the inspiration for this paper, but in contrast to the approaches reviewed above, a non-linear model of material behaviour must be used throughout
the deformation, to design a realistic sheet forming process. Thegeometrical and material nonlinearities of metal forming prevent development of exact analytical models and render existing computational methods (such as the ubiquitous finite element method) too slow for solving such a loosely constrained inverse problem. However, the definition of optimal forming can be applied to find optimal inverse (or "un-forming") processes (with
minimum effective strain) subject to appropriate constraints, and the stress history of the reverse of this (the proposed forming process) then found from a suitable finite element model. The ambition of this paper is thus to assemble these component methods for the first time to explore process design by optimally "un-forming" a final part to an unspecified flat blank, without prior
specification of the process.

Proposed new method for process design by "un-forming"

The algorithm proposed in this paper is summarised in Fig. 1: a target part is optimally "un-formed" (measured by the integral of
total effective strain over the workpiece) to a flat blank, subject to kinematic constraints. An initial set of constraints is used to specify a "forming strategy" (such as incrementally reducing the curvature of the part, or specifying a direction of motion for the part perimeter) and these constraints are iteratively refined based on predictions of the boundary conditions that must be applied to the workpiece during forming. An additional set of constraints is used
to prescribe desired deformation characteristic (such as pure shear deformation).

The method shown in Fig. 1 is iterative: for some set of process constraints, a target part is optimally "un-formed" to a flat blank; the deformation history of this un-forming is then reversed to calculate the strain and stress histories of the forwards forming process that would form the target part from the calculated flank blank. From these stresses, the boundary conditions required to establish equilibrium are calculated. A user can then examine the feasibility of applying these boundary conditions and iteratively refine the constraints applied to the next un-forming calculation. The feasibility potential can be assessed by the strains involved in the deformation and the external load required to produce them. When the resulting deformation is revealed to be infeasible, the
original un-forming strategy and assumptions are amended and the process is repeated, until a feasible deformation path is found.

Further details of these two stages of the calculation follow in the next two sections.

Optimally un-forming a target part subject to constraints The algorithm of Fig. 1 could be used for any achievable target

part geometry, but in this paper only a square cup shape is con-
sidered, shown in Fig. 2. This geometry was chosen, based on the part classification work of [23] since it contains both developable and non-developable regions, as well as flanging. Furthermore, a process that achieves this geometry locally can potentially achieve similar or increasingly complicated shapes by superposing appropriate toolpaths. The cup is 40 mm deep with a total width of 120 mm. The rim on each side is 10 mm wide. A triangular

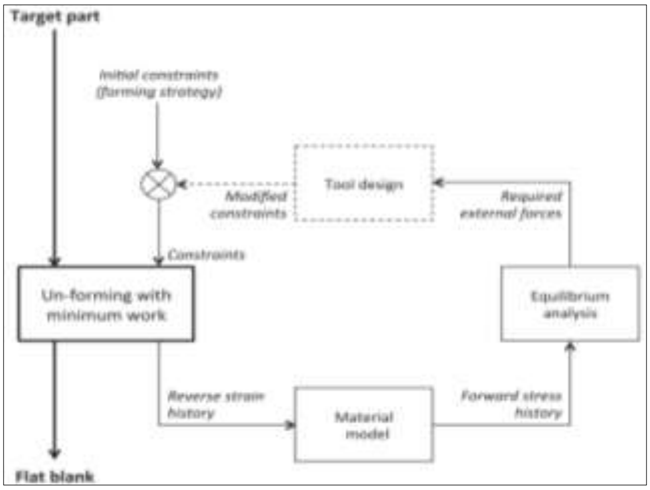

Fig 1: Block diagram showing the basic steps in the suggested method. (Dashed lines indicate stages of the algorithm currently performed manually

mesh is created on this part – also shown in Fig. 2 – to allow for discretization of the constraints and the objective function that follow. It is assumed that the process has sufficient symmetry that only one quarter of the square cup need be analysed in the simulation, in order to reduce computational time. The un-forming optimisation algorithm minimises the plastic work done, W, on the entire domain of the workpiece, Ω, at each step of un-forming. The nodal coordinates are represented by x. Chung and Richmond [21, 22] call this "ideal forming" since, under certain conditions, it distributes deformation across the workpiece so that deformation before failure is maximised. Thus, the objective function maximises the feasible deformation globally. Specifically, the objective function f is defined as:

$$\arg\min_{x\in\Omega} f = \sum_{i=1}^{m} dW_i$$

$$\text{subject to } A_i^{t_k} = A_i^{t_0},\ \forall\ i\in\Omega$$

$$x_p^{t_k} = C,\ \forall\ \in\omega,\ \omega\in\Omega$$

and to symmetry conditions across the x and y-axis. Here, m is the number of elements in the mesh. The area of element i is A i while the superscripts indicate the time steps; for each element the area at any time step is constrained to remain equal to its initial area. For any sheet material discretised in two dimensions and under
the constant volume hypothesis for plastic deformation of metals, this implies that there is no change in thickness or equivalently that the deformation is in pure shear. The variables in the optimisation are the Cartesian coordinates of all the nodes in the mesh, since they are the basis for calculating both the plastic work decrement and the updated areas of the elements. The additional kinematic constraints applied in each method are described separately in each relevant section below, and are similarly defined
in the code in terms of the nodal coordinates.

They are noted above in a generic form with the second constraint. The plastic work in each element, at each step, is given [24] as:

(4) dW = σ¯d ε¯dV

where σ¯is the flow stress, d ε¯is the equivalent plastic strain decrement and dV is the element volume. The minimisation occurs after each displacement-controlled step mentioned in the constraints above.

The elements are treated as constant strain triangles and an equivalent plastic strain decrement for each element can be calculated at each step using the natural strain on each element edge.

where x j and x k are the node coordinates that define edge h of
each element. The three strains are combined in one vector. Effectively, the 2D element does not account for bending energy, but given the required large changes in Gaussian curvature for general un-forming, stretching energy is expected to dominate. From these strains their Cartesian counterparts for each element, i, are obtained: with

$$de_i^{t_k} = \begin{bmatrix} de_{xx} \\ de_{yy} \\ 2de_{xy} \end{bmatrix} = T_e \epsilon^{t_k}$$

with

$$T_e = \begin{bmatrix} c_1^2 & s_1^2 & s_1 c_1 \\ c_2^2 & s_2^2 & s_2 c_2 \\ c_3^2 & s_3^2 & s_3 c_3 \end{bmatrix}$$

the "strain gauge rosette" transformation tensor. Here, c and s are sines and cosines of the element angles to a local Cartesian coordinate system in the plane of the element. The node coordinates are transformed to this local system.

The equivalent plastic strain decrement for the element is given by these Cartesian strains as:

In the examples, the material parameters, σ y and n, were defined for aluminium 1100-O as 172 MPa and 0.25 respectively, based on values taken from [25], and adapted to exclude the elastic region. The form of Eq. (9) maintains the same values for σ y and n as the referenced source. Effectively, only the plastic strains are taken into account since they are assumed to be much greater than the elastic strains. Currently the method employs an isotropic model, but with some additional work, the formulation can in-
clude anisotropic materials. Substituting Eqs. (8) and (9) in Eq. (4), and summing over all elements as per Eq. (1), the plastic work required at the current step can be calculated and thus it can be supplied to the optimizer. Both the optimisation process and the inclusion of constraints – in these examples the constant area constraint – are performed via the well-

known ‘fmincon’ function with the ‘interior-point’ algorithm in MATLAB [26, 28]. Specifically, the ‘interior-point’ algorithm runs until an objective function tolerance is achieved, or the variable tolerance is reached, etc. MATLAB documentation explains additional stopping criteria. Table 1 gives the options chosen for ‘fmincon’.

The optimal decrement is subtracted from the total effective strain at each step. After each step the resulting nodal coordinates post-processing. The cumulative equivalent plastic saved to be used in the next optimisation step. The each step, and for each triangle is also calculated equivalent plastic strain estimate.

are saved for strain is also flow stress at based on the Estimating the initial plastic strains Since the part is un-formed, the accumulated plastic strain at the start of the calculation above, must be initialised at some va- lue, such that after it has decreased during un-forming, the final flat workpiece has no plastic strain. To initiate the calculation this trial value of accumulated plastic strain must be estimated. Hence, the assumption is made that for any forming operation in this context, the flat blank has no initial plastic strains. But for the starting formed part in the un-forming method, the plastic strains need to be estimated. This estimate is obtained with the following iterative method: The simulation is initialised with a value of $\bar{\varepsilon} = \bar{\varepsilon}(01) = 0.5$ throughout the workpiece. The un-forming strategy is followed to completion and the final equivalent plastic strains on the flat workpiece are recorded. The final $\bar{\varepsilon}_f$ improves the initial guess by using:

with r being the current step in the iteration. The process is repeated using the updated initial plastic strain. After a limited set of steps, a mean final strain is obtained that is sufficiently close to zero, allowing for the initial strain field to be verified. To facilitate this, the material law is slightly adapted to ensure only positive values of $\bar{\varepsilon}_E$ are used:

This modification prevents unphysical negative equivalent strains from affecting the result and encourages convergence of the iteration.

Iteratively refining the process constraints Once the optimisation algorithm has completed, to create a flat workpiece, a complete history of displacement in un-forming has been established, so this can now be reversed and considered to be the history of forming the part from the blank. For each displacement-controlled forward step, an incremental strain field can be calculated by comparing nodal coordinates for successive time steps, as defined in Eq. (10). A stress field can then be produced through the flow rule $de_{ij} = \lambda \sigma'_{ij}$, where σ'_{ij} represents the deviatoric stresses and λ is the plastic parameter given from the flow stress curve as:

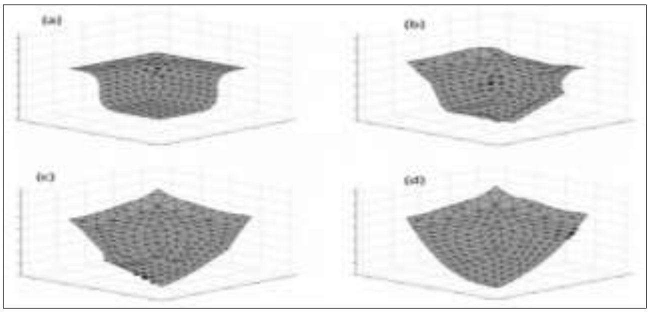

Fig 2: Progress of the increasing GC local flattening method. The black dot and the grey dots represent the node currently being flattened along with its neighbours

Results

Each simulation produces the geometry of the mesh at each step, which can be reversed and animated to illustrate the deformation path of the candidate new process.

As previous work has focused on the blank shape that can be inferred from flattening, a corresponding shape is found at the end of each simulation here.

Once the geometry of the mesh at each step has been obtained, strain and stress fields are produced following the method of Section 3. The strain field is visualised by plotting vectors representing the principal strains on each element.

The results for the forming strategies discussed above are shown in this section, but first the validation of the method through the test case example is given.

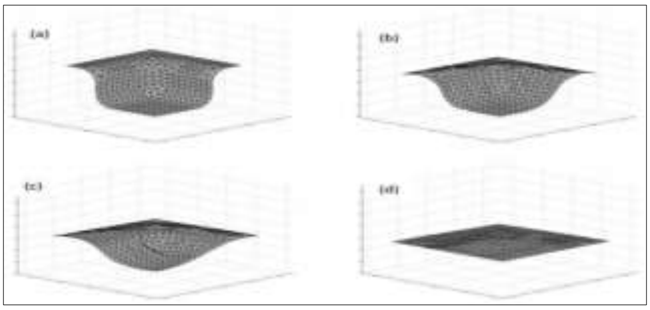

Fig 3: The deformation path for the radial boundary puli strategy. Frames 2, 58, 98 and 130 are shown.

Results for un-forming via highest negative GC

The deformation trend is presented in Fig. 7, where few frames from the output (specifically frames 2, 102, 202 and 302 out of 302) are shown. The node currently being flattened in each frame is marked with a black dot, along with its neighbours in grey. This method succeeds in gradually reducing the magnitude of Gaussian Curvature across the workpiece but fails to find a continuous deformation path to the global flattened state: the end point of this strategy is the smooth ‘bowl’ shape in Fig. 7(d).

The algorithm using this forming strategy was computationally expensive, since to maintain the co-planarity constraint, a number of matrix operations must be performed at each iteration of the optimisation. Perhaps for the same reason, ‘fmincon’ frequently reaches an infeasible solution. This result also prevented the iterative process to estimate the initial equivalent plastic strain field.

Results for un-forming via radial boundary pull

This strategy provides deformation data that are more applicable to a process design. “Pulling” the periphery of the cup along a linear trajectory forces the rest of the workpiece to deform in smooth and controlled manner. The resulting stages of deformation are shown in Fig. 8. The algorithm does not run to completion since the required pull at the periphery needed to produce the flat shape is less than the length of the trajectory described in the method. Instead, by step 65 out of 100, which corresponds to frame 130, all nodes are within 3mm of each other in the z-direction. This plane is not the xy plane, but instead a parallel plane above it; during the deformation, the centre of the workpiece slowly rises. The method notably produces the most complete unforming and the most intuitive intermediate geometries.

The effective plastic strain for each element is calculated from Eqn (10) and shown in Fig. 9 (left). The results are shown halfway through the flattening process – i.e. at frame 66. In this example,the greatest deformation occurs where the brim of the cup meets the corners. The the regions in the deeper part of the cup remain mostly unaffected during this stage of the deformation.

For some elements, the effective strains are potentially exceeding the ultimate strain for the material defined here. This would indicate that given the chosen constraints the process is not applicable to the specific part. A more complicated kinematic strategy could dissolve the concentration of strains in particular
locations.

Similarly, the deviatoric stresses are calculated using Eq. (14) and also plotted in Fig. 9 (right). Again, this is useful in observing the relative stress state of elements or groups of elements and examining its feasibility.

Conclusion

This paper has made a first demonstration of the transfer of a technique from the garment industry to sheet metal forming. The approach allows un-forming to a flat blank, while minimising work and maintaining sheet thickness, and allows for any forming strategy, three of which have been demonstrated. The method produces strain fields which can be used to assess the feasibility of forming a part subject to specified process constraints and for given material parameters.

A wide range of forming strategies could be explored in future, including prescribed thickness variation for the workpiece. Similarly, un-forming can be instigated at different locations, as demonstrated in the Gaussian Curvature strategy, but could also focus on other metrics, for example increasing Mean Curvature.In addition to the direct calculations on the elements above, it is also possible to use the deformation path of the mesh to produce the exact forces required on each node.

This can be demonstrated through use of a standard FEM package, by setting up a displacement-controlled simulation that corresponds to the results of the method of Section 3.1 with the displacements are provided in the forwards direction – from flat geometry to the final target part.

For example, in Fig. 11, the halfway frame in the simulation is shown, with a profile view of the workpiece and the necessary nodal forces. The largest forces are needed in shaping the flange at this time step while the bottom and walls of the cup only require relatively small tool forces.

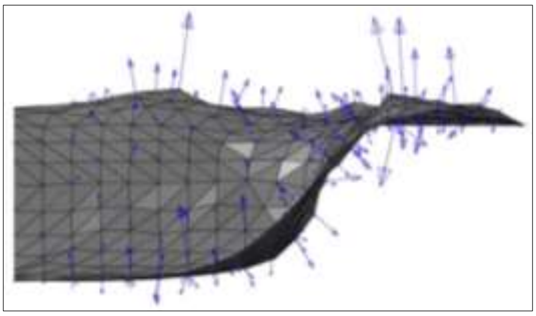

Fig 4: A relative plot in profile view of nodal forces required halfway through the process to produce the desired deformation.

This work has presented a design strategy that can be employed for a variety of process constraints, and potentially to a wide array of challenging target parts. A general optimisation solver has been used in this work, but the method could be improved in future work with faster algorithms – see for example Raithatha [32]. The high variability in the direction of nodal forces in Fig. 11 is a consequence of the pure shear constraint imposed throughout this deformation. Future work will aim to reconcile the predicted forces to available tooling options, through iterative refinement of the process constraints to be more representative of the tractions that could be applied by real tools, as indicated in Fig. 1. This could include use of template forms of nodal force that could exist under a tool, and a clustering algorithm to classify regions of similar required forces.

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