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Reduced-order distributed fusion with application to object tracking

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Abstract
In this paper, we propose a novel reduced-order track-to-track fusion filter (ROF) for estimating not all state variables, but only those variables that indicate useful information of a target system for control. The ROF algorithm is designed for multisensory continuous-time stochastic systems. Its communication loads and computational complexity are not so complicated due to usage of the reduced-order local Kalman filters. Performance of the ROF and its estimation accuracy using the covariance intersection fusion are demonstrated on a 2D motion model with several GPSs. Comparative analysis of the ROF with the global optimal centralized Kalman filter is presented. Simulation results demonstrate practical effectiveness of the proposed ROF.

Keywords: state estimation, multisensory system, kalman filtering, reduced-order filter, covariance intersection

Introduction

Many advanced estimation and target tracking systems, including aerospace, defense, robotics, automation systems and others involve multiple homogeneous or heterogeneous sensors that are spatially distributed to provide a large coverage, diverse viewing angles, or complementary information. Up to now, there exist two basic information fusion methods: the centralized fusion and the distributed fusion, depending on whether the raw data are used directly for fusion or not [1]. In centralized fusion, all raw measurements are sent to the fusion center, while in distributed fusion, each sensor only sends in the processed data (local estimates) for their subsequent combing or weighting. Centralized fusion, despite of its heavy computation and communication burden at the fusion center and poor survivability, can provide globally optimal state estimation. Distributed fusion, usually gives a suboptimal estimate. However, it has faster real-time processing and stronger fault-tolerance abilities. A system with multiple distributed sensors has many advantages such as increasing of capability, reliability, robustness, and survivability of the system. In a distributed configuration system each sensor has its own local processor and track. If all tracks represent the same target, then one can combine (fuse) the corresponding estimates. If done appropriately, the distributed fusion will yield more accurate estimates than single sensor based estimates [2].

Based on the distributed approach the fusion estimate $\hat{x}_t^{fus} \in \mathbb{R}^n$ of the state vector $x_t \in \mathbb{R}^n$ represents the weighted sum of the local state estimates,

$$\hat{x}_t^{fus} = \sum_{s=1}^{L} A_t^{(s)} \hat{x}_t^{(s)}, \ \sum_{=1}^{L} A_t^{(s)} = I_n, \qquad (1)$$

Where $\hat{x}_t^{(s)} \in \mathbb{R}^n$ is the local estimate, $A_t^{(s)} \in \mathbb{R}^{n\times n}$ is the weight, $I_n \in \mathbb{R}^{n\times n}$ is the identity matrix, and L is the number of sensors.

The optimal weights $A_t^{(s)} = A_t^{(s)}\left(P_t^{(ij)}\right)$ depend on the cross-covariance (CC) of the local tracks (estimates), $P_t^{(ij)} = Cov\left(\hat{x}_t^{(i)}, \hat{x}_t^{(j)}\right)$, $i \neq j$, that can be found in [3-6]. To implement the optimal fusion estimator (1) we need to calculate all CCs, $P_t^{(ij)}$, $i, j = 1, \dots, L$, and perform many matrix multiplications on matrices of large size. Although measurements at different local sensors have independent noises, the local estimates are correlated due to the common process noise of a target motion, therefore in general $P_t^{(ij)} \neq 0$. However calculation of all CCs, $L(L-1)/2$, requires a lot of information to be transferred between sensors which drastically complicates implementation of the optimal fusion (1).

Ignoring the CCs will lead to filter divergence. To overcome this divergence problem of the filter (1), the CCs should be taken into account in either explicit or implicit manner. Instead of striving for the optimal fusion, there are different suboptimal fusion techniques fully or partially removing the cross-covariance information. The prime example of such techniques is the Covariance Intersection (CI) [7] and its modifications [8-13]. They propose to suboptimally fuse the local estimates $\hat{x}_t^{(1)}, \dots, \hat{x}_t^{(L)}$ with scalar weights (CI's indices) without assuming any knowledge on the CC between tracks. However the suboptimal fusion based on the CI's idea is not sufficient to guarantee good fusion results, because the uncertainty between the local tracks is underestimated.

Considering the drawbacks of the existing fusion techniques, we propose a new low-complexity reduced-order fusion method to fuse some components of a state vector being referred to as required state components (RC). This method that includes only non-zero CCs between the RC, improves accuracy of the fusion result compared to the existing suboptimal fusion, and also its communication loads and computational complexity are not so complicated due to usage of the reduced-order local filters for the RC. The utilization of RC has a practically useful motivation when a fusion estimate of the components is computed at the fusion center. For example, if an object's state vector $x_t =$

$(p_t, v_t) \in \mathbb{R}^6$ consists of the position $p_t = (x_{1,t}, x_{2,t}, x_{3,t})$ and the corresponding velocity $v_t = (\dot{x}_{1,t}, \dot{x}_{2,t}, \dot{x}_{3,t})$ components, then $p_t \in \mathbb{R}^3$ becomes the RC in case where the control law depends only on p_t, i.e. $u = u(p_t)$.

The paper is organized as follows. Problem formulation is given in Section 2. A low-complexity reduced–order filtering algorithm is presented in Section 3. A numerical example is given in Section 4. The conclusions are presented in Section 5.

Problem Formulation

Consider a continuous-time linear system with L local sensors,

$$\begin{aligned} \dot{x}_t &= F_t x_t + G_t \xi_t,\ t \geq 0, \\ y_t^{(i)} &= H_t^{(i)} x_t + \eta_t^{(i)}, \quad i = 1, \dots, L, \end{aligned} \tag{2}$$

where $x_t \in \mathbb{R}^n$ is the state vector, $y_t^{(i)} \in \mathbb{R}^{m_i}$ is the measurement vector from the ith sensor, $\xi_t \in \mathbb{R}^q$ and $\eta_t^{(i)} \in \mathbb{R}^{m_i}$ represent zero-mean white Gaussian noises with covariances $Cov(\xi_t, \xi_s) = Q_t \delta_{t-s}$ and $Cov(\eta_t^{(i)}, \eta_s^{(i)}) = \delta_{t-s} R_t^{(i)}$, respectively, and δ_t is the Dirac delta function. We also assume that the initial state $x_0 \sim \mathbb{N}(m_0, P_0)$, process noise ξ_t, and measurement errors $\eta_t^{(1)}, \dots, \eta_t^{(L)}$ are mutually uncorrelated.

We are interested in the fusion estimation not all components $x_1, \dots, x_n$, of the state vector but only ℓ of them ($\ell \leq n$), using the overall sensor measurements $Y_0^t = \{y_s^{(1)}, \dots, y_s^{(L)}: s \in [0, t]\}$. As we already mentioned, the part of ℓ components represents the RC. Without loss of generality it is possible to consider the first ℓ components $x_1, \dots, x_\ell$ of a state vector as the RC, $U_t \in \mathbb{R}^\ell$, i.e.,

$$x_t = \begin{bmatrix} U_t \\ V_t \end{bmatrix} \in \mathbb{R}^n,\ U_t = \begin{bmatrix} x_{1,t} \\ x_{2,t} \\ \vdots \\ x_{\ell,t} \end{bmatrix},\ V_t = \begin{bmatrix} x_{\ell+1,t} \\ x_{\ell+2,t} \\ \vdots \\ x_{n,t} \end{bmatrix},\ t > 0. \tag{3}$$

So, our goal is to find the optimal fusion filter (estimator), $\widehat{U}_t^{ROF}$, for the RC $U_t \in \mathbb{R}^\ell$.

Remark: At $\ell < n$ we have the reduced-order fusion filter («ROF»).

According to the distributed approach the fusion estimate $\widehat{U}_t^{ROF}$ represents the following weighted sum,

$$\widehat{U}_t^{ROF} = \sum_{i=1}^{L} W_t^{(i)} \widehat{U}_t^{(i)},\ \sum_{i=1}^{L} W_t^{(i)} = I_\ell, \tag{4}$$

where $\widehat{U}_t^{(1)}, \dots, \widehat{U}_t^{(L)} \in \mathbb{R}^\ell$ are the local estimates of the RC U_t in (3) with the corresponding unknown lower dimension $\ell \times \ell$ matrix weights $W_t^{(1)}, \dots, W_t^{(L)}$.

The minimum mean square error (MMSE) approach is applied to find the optimal weights.

Further, we present the novel low-complexity ROF for the linear multisensory system (2).

Low-Complexity ROF for Linear Multisensory Systems

The local Kalman estimate $\hat{x}_t^{(i)}$ of the state x_t and the corresponding covariance $P_t^{(ii)} = Cov(\hat{x}_t^{(i)}, \hat{x}_t^{(i)})$ are calculated using the continuous Kalman filter (KF) equations [14, 15],

$$\begin{cases} \dot{\hat{x}}_t^{(i)} = F_t \hat{x}_t^{(i)} + K_t^{(i)} \big(y_t^{(i)} - H_t^{(i)} \hat{x}_t^{(i)}\big),\ \hat{x}_0^{(i)} = m_0, \\ \dot{P}_t^{(ii)} = F_t P_t^{(ii)} + P_t^{(ii)} F_t^T + \tilde{Q}_t,\ P_0^{(ii)} = P_0, \\ K_t^{(i)} = P_t^{(ii)} H_t^{(i)^T} R_t^{(i)^{-1}},\ \tilde{Q}_t = G_t Q_t G_t^T. \end{cases} \tag{5}$$

Then we have the following result.

Theorem («MMSE Weights for ROF»). *For the linear multisensory system (2) the optimal MMSE weights $W_t^{(1)}, \dots, W_t^{(L)}$ for the reduced-order fusion filter (4) can be explicitly written down as*

$$\mathbb{W}_t = (\mathbb{I}_\ell^T \mathbb{C}_t^{-1} \mathbb{I}_\ell)^{-1} \mathbb{I}_\ell^T \mathbb{C}_t^{-1},$$
$$\mathbb{C}_t = \begin{bmatrix} C_t^{(11)} & \cdots & C_t^{(1L)} \\ \vdots & \ddots & \vdots \\ C_t^{(L1)} & \cdots & C_t^{(LL)} \end{bmatrix} \in \mathbb{R}^{\ell L \times \ell L} \tag{6}$$

where

$$\mathbb{W}_t = [W_t^{(1)} \dots W_t^{(L)}] \in \mathbb{R}^{\ell \times \ell L},$$
$$\mathbb{I}_\ell = [I_\ell \cdots I_\ell]^T \in \mathbb{R}^{\ell L \times \ell}.$$

Here the covariance $C_t^{(ii)}$ and cross-covariance $C_t^{(ij)}$ represent the $\ell \times \ell$ submatrices lying in the upper left corner of $P_t^{(ii)}$ and $P_t^{(ij)}$, respectively. Moreover $P_t^{(ii)}$ satisfies (5) and $P_t^{(ij)}$ is described by the Riccati-like differential equations [16],

$$\begin{aligned} \dot{P}_t^{(ij)} &= \big(F_t - K_t^{(i)} H_t^{(i)}\big) P_t^{(ij)} + P_t^{(ij)} \big(F_t - K_t^{(j)} H_t^{(j)}\big)^T \\ &\quad + \tilde{Q}_t,\ P_t^{(ij)} = P_0,\ i, j = 1, \dots, L,\ i \neq j. \end{aligned} \tag{7}$$

Proof of Theorem. The MMSE criterion and the ROF covariance take the form

$$\begin{aligned} &\min_{\mathbb{W}_t \mathbb{I}_\ell = I_\ell} \mathbb{E} \big\| U_t - \widehat{U}_t^{ROF} \big\|^2 = \min_{\mathbb{W}_t \mathbb{I}_\ell = I_\ell} P_t^{ROF},\ P_t^{ROF} = \mathbb{E}[e_t^{ROF} (e_t^{ROF})^T], \\ &P_t^{ROF} = \mathbb{E}\Big[\textstyle\sum_{i,j=1}^{L} W_t^{(i)} e_{u,t}^{(i)} \big(W_t^{(j)} e_{u,t}^{(j)}\big)^T\Big] = \mathbb{W}_t \mathbb{C}_t \mathbb{W}_t^T,\ e_{u,t}^{(i)} = U_t - \widehat{U}_t^{(i)}, \end{aligned} \tag{8}$$

Where $\mathbb{C}_t \in \mathbb{R}^{\ell L \times \ell L}$ is the block matrix, and $C_t^{(ij)} = Cov(\widehat{U}_t^{(i)}, \widehat{U}_t^{(j)})$.

The Lagrange function $\mathcal{L}_k$ is defined as

$$\mathcal{L}_t = tr(\mathbb{W}_t \mathbb{C}_t \mathbb{W}_t^T) + tr[\Lambda(\mathbb{W}_t \mathbb{I}_\ell - I_\ell)], \tag{9}$$

Where $\Lambda \in \mathbb{R}^{\ell \times \ell}$ is the Lagrange multiplier.

Differentiate (9) with respect to $\mathbb{W}$ and setting the obtained result to be zero we get

$$\frac{\partial \mathcal{L}_t}{\partial \mathbb{W}_t} = 2\mathbb{W}_t \mathbb{C}_k + \Lambda^T \mathbb{I}_\ell^T = 0. \tag{10}$$

Combining (10) with the constraint $\mathbb{W}_t \mathbb{I}_\ell = I_\ell$ we obtain the explicit solution,

$$\mathbb{W}_t = (\mathbb{I}_\ell^T \mathbb{C}_t^{-1} \mathbb{I}_\ell)^{-1} \mathbb{I}_\ell^T \mathbb{C}_t^{-1},\ \frac{1}{2}\Lambda^T = -(\mathbb{I}_\ell^T \mathbb{C}_t^{-1} \mathbb{I}_\ell)^{-1}. \tag{11}$$

This completes the derivation of (6).

Numerical Example: 2D Object Tracking with Random Velocity

In this section, the performance of the proposed ROF is evaluated for a linear dynamical model with multisensory

measurements.
Consider a nearly constant velocity model which is given by

$$\dot{x}_t = Fx_t + G\xi_t,\ t \in [0,1],\ x_0 \sim \mathbb{N}(m_0, P_0),$$
$$F = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},\ G = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix},\ \xi_t \sim \mathbb{N}(0, Q). \quad (12)$$

Here $x_t \in \mathbb{R}^4$ is the state of the object being defined as $x_t^T = [p_t^T, v_t^T]$, where $p_t = [x_{1,t}, x_{2,t}]^T$ and $v_t = [x_{3,t}, x_{4,t}]^T$ denote the Cartesian position and the corresponding velocity on the horizontal plane, and $\xi_t = [\xi_{1,t}, \xi_{2,t}]^T$ is a white noise being used to model unknown unwanted accelerations, turbulence, wind disturbance and so on, with an appropriate intensity matrix $Q = diag(q_1, q_2)$, which is a design parameter.
Assume that GPS sensors collect information about the object position p_t. Then the multisensory measurement model takes the form,

$$y_t^{(i)} = \begin{bmatrix} y_{1,t}^{(i)} \\ y_{2,t}^{(i)} \end{bmatrix} = H^{(i)} x_t + \begin{bmatrix} \eta_{1,t}^{(i)} \\ \eta_{2,t}^{(i)} \end{bmatrix},\ H^{(i)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$
$$i = 1, \dots, L, \quad (13)$$

Where $\eta_t^{(i)}$ is a white noise with an intensity $R^{(i)} = diag(r_1^{(i)}, r_2^{(i)}),\ i = 1, \dots, L.$

Our interest is in fusing the estimates of the object position only. In this case the position is chosen as the RC, $U_t = p_t \in \mathbb{R}^2$ with $\ell = 2$, and the velocity components are ignored in the fusion process. The position $p_t \in \mathbb{R}^2$ is estimated using the centralized KF (CKF), and two fusion filters ROF Eq.(4) with $\ell = 2$, and the CI filter [7], respectively, for the purpose of comparison.

Scenario A
Here we compare three filters CKF, ROF and CI including both identical and different sensors with $L = 3$. The values of model parameters, noise covariances and initial conditions are selected as below:

$$m_0 = [10, 10, 2, 2]^T;\ P_0 = I_4;\ Q = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix};$$
$$R^{(1)} = R^{(2)} = R^{(3)} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix};\ R^{(4)} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}. \quad (14)$$

We consider the effect of replacing one sensor with a more accurate one. In Fig.1, the mean-square error (MSE), $J_t = \mathbb{E}\|p_t - \hat{p}_t\|^2$, is depicted for two measurement cases: the first includes three identical sensors $R^{(1)} = R^{(2)} = R^{(3)}$ (Case 1), and in the second one the third sensor $R^{(3)}$ is replaced with a more accurate one $R^{(4)} \ll R^{(1)} = R^{(2)}$ (Case 2).

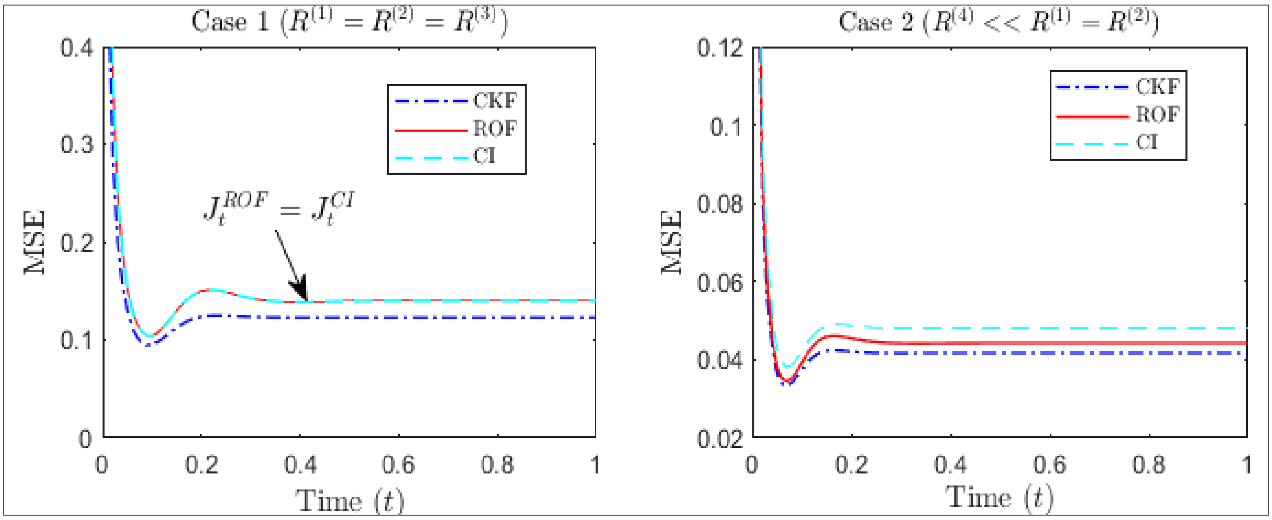

Fig 1: Comparison of MSEs for 2D motion using CKF, ROF and CI filter.

Comments on Fig.1 are in order.

- The MSEs in Case 2 are less than in Case 1 for all filters. That confirms the natural fact that replacing the third sensor $(R^{(3)})$ with a more accurate one $(R^{(4)} \ll R^{(3)})$ decreases the estimation error.
- In Case 1 the MSEs J_t^{ROF} and J_t^{CI} are equal due to the fact that the weight matrices in the are equal for identical sensors.
- In Case 2 the relative errors $\Delta J_t(\%) = |(J_t - J_t^{CKF})/J_t^{CKF}| \times 100\%$ for the ROF and CI are $\Delta J_t^{ROF} = 5.9\%$ and $\Delta J_t^{CI} = 15.2\%$ for $t > 0.2$, respectively. The difference between J_t^{CKF} and J_t^{ROF} is negligible, i.e., $J_t^{CKF} \approx J_t^{ROF}$. It is shown that the ROF is slightly worse than the CKF, but it computational complexity is lower than of the CKF.

Scenario B
In scenario *B* we compare the performance of the CKF, ROF and CI through the Monte Carlo simulation with the three sensors $R^{(4)} \ll R^{(1)} = R^{(2)}$ for Case 2. The estimated trajectories $\hat{p}_t = (\hat{x}_{1,t}, \hat{x}_{2,t})$ are plotted for the three filters in Fig.2. The average MSE for position over the time interval $t_k \in [t_{k_1}, t_{k_2}]$ is defined as

$$\bar{J}[t_{k_1}, t_{k_2}] = \frac{1}{k_2 - k_1 + 1} \sum_{k=k_1}^{k_2} \|p_{t_k} - \hat{p}_{t_k}\|^2, \quad (15)$$

Where $p_{t_k} = (x_{1,t_k}, x_{2,t_k})$ is the reference (true) trajectory.
Fig.2 shows that the CKF and ROF trajectories are quite close, and both are better than the CI trajectory. This is also confirmed by the values of the average MSEs, $\bar{J}^{CKF} = 0.0414$, $\bar{J}^{ROF} = 0.0419$, and $\bar{J}^{CI} = 0.0474$ within the interval $t_k \in [0,1]$. So it is clearly seen that the ROF for the 2D model performs quite comparable to CKF.

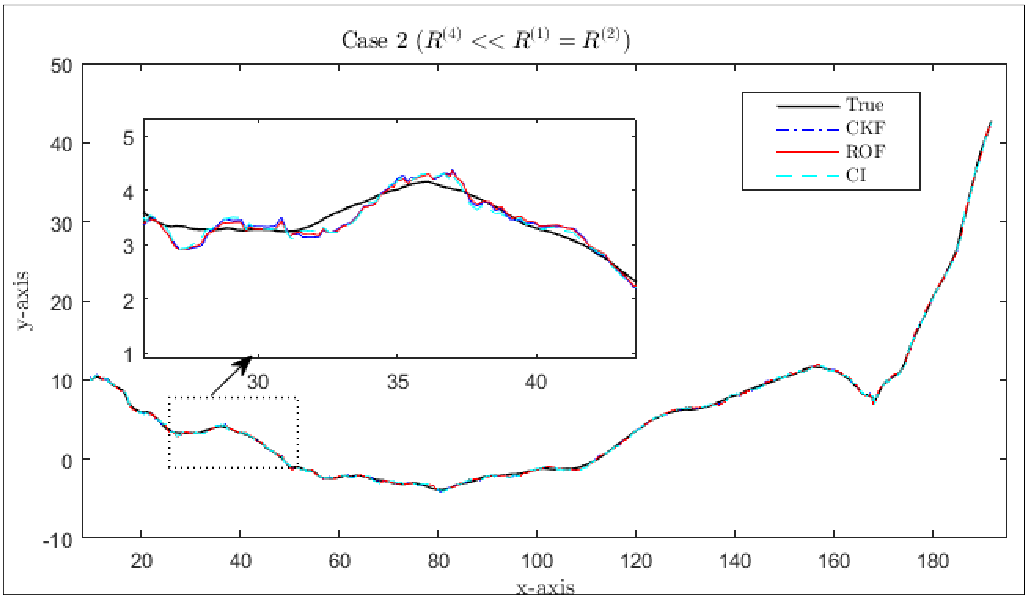

Fig 2: Trajectories of 2D motion using CKF, ROF and CI with 3 sensors.

Conclusions

A novel reduced-order distributed fusion filter (ROF) for linear continuous-time multisensory dynamical systems is proposed. The ROF represents a weighted sum of local Kalman filters for RC. The optimal ROF's weights are determined by the MMSE criterion.

The main contributions are listed in the following.

1. Using the mean square estimation approach, formulas for the optimal ROF's weights are derived.
2. To verify the effectiveness of the ROF, it is implemented on a multisensory 2D motion model. Through this implementation, it is found that the ROF and CKF perform quite comparably. However, the ROF algorithm is more effective than the CKF in terms of computational complexity. The CI filter is much worse than the ROF, despite its computational simplicity due to ignoring all CCs. Performance of the proposed ROF illustrates its practical usefulness and suitability for multisensory systems.

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