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Analysis of field size distribution by lognormal function

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Abstract
In managing uncertainty and quantifying risk in basing exploratory decision making on investment drilling program the knowledge of available data on the likelihood sizes of discovered fields and minimum speculative capital provide some sense of certainty of success. The truth is that making decisions on risky ventures involves more than looking at the average rate of return or the average present worth. The number of ventures that can reasonably be undertaken to guarantee chance of success of each venture and the magnitude of a success was analyzed by log normal transformation of available sizes from similar fields.
The log normal is an important probability function from the view point of distribution of natural resources, such as volumes of petroleum reservoirs as they occur within geologic provinces. The highly skew nature of this distribution indicates that the logarithm of the variable is normally distributed.
This paper summarizes the development and application of appropriate frequency distribution for possible assumptions and final outcomes will encourage management decisions. Thus, any group venturing into searching for oil and gas requires assurances of making money. It is all about being sure of making money on the average an in fact we must be fairly certain of making profit from a limited investment.

Keywords: investment, ventures, skew, drilling, risk

Introduction

We gain a lot of mileage by treating an exploration program as comprising of a series of drilling attempts based on limited and imperfect knowledge from similar and related ventures even under geological and geophysical leads (Arps, 1993) [1].
For our simplified analysis we used the same cost for each attempt for an exploration attempt might be the cost of drilling a well, or the cost of testing a formation in a borehole (Magee and John F, 1964) [2]. The chance of making a profit from an exploratory program depends on the sizes likely of be find in any attempt (Newendorp, P. D.,1967) [3]. This chance is analyzed by log-normal probability distribution of field sizes in a given region. In our evaluation we chose to see if an exploration program has a 'reasonable' chance of making a certain rate of return, R, or some positive present worth W (Walstrom, J. E.,1967) [4].
Since the program may result in from 0 to N discoveries, as governed by the chance of each attempt, the number of attempts and the laws of probability, we must consider the economics of each possible result in computing the present worth. For an exploratory program to make this rate of return, R, from exactly x discoveries must equal the cost of exploration. Certain aspects of many of the risk management standards have provided some measurable improvement on risk, by which the confidence in estimates and decisions seem to increase (Hayward J. T., 1963) [5].

Methodology

The price that an economic unit can safely pay for an investment must be consistent with the objectives of the unit. Capitals employed by industrial enterprises returns products and services, as well as dividends and sufficient return cash flow for development and expansion. These things are ultimately returned to the environment. In final analysis it must be the environment and interaction in the environment that provide the standards by which an economic venture will be judged.
It is a desperate misconceived assumption that the profit motive controls business enterprises. Profit is necessary for wages, for the necessary increase in capital, for incentives; but, it is the fear of loss that dominates the business complex and the non-profit organizations. This principle can be traced to the concept that the measure of an organization's efficiency is its continued survival. In many situations, there will be other factors present besides survival and profit. For example, an oil company might be more interested in maximizing its reserves of crude oil. Investments must be limited and distributed so as to assure survival at an acceptable level through a series of ventures; then the prospective investments may be further scrutinized to choose those that will best satisfy other objectives.
Statistical decision methods can be helpful in assuring continued existence at an acceptable level. These methods are widely used in many areas of physical, biological and operations research. In general, a predetermined risk is set as a standard for guidance. As far as drilling exploratory oil wells is concerned, it is conceivable that a very long sequence of dry holes could be encountered and unless some risk is accepted, no exploratory wells could be drilled.
What cannot be avoided should be managed. The acceptance of a 5 percent risk has been assumed for the examples. Other levels of risk could be used in exactly the same way.
Statistical decision methods can be applied to the distributions of cumulative present values of a group of ventures, with respect to the number of ventures in the group. Normal plot of available data will be used to show the parts of the probability distribution that are used in formulating a statistical decision. The solution for the number of ventures in the group that will insure 95 percent confidence of success occurs where the line representing the required minimum

return intersects the 95 percent line. If there are more ventures in the group, then the chances of success will be greater; less than this number involves a risk greater than 5 percent.

A statistical solution for the appraisal value for the statement that how many number of attempts n will give the required confidence of success is given by the following equation, and will be developed as follows.

Mathematical Description

The log normal distribution is described for a random variable $x = \ln(y)$, where variable x is normally distributed and is a logarithm function of variable y, and the probability distribution functions relating the two variables is

$$f(y)dy = g(x)dx \tag{1}$$

$$g(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}, -\infty \leq x \leq \infty \tag{2}$$

Thus equation 1 can be written for $f(y)$ as

$$f(y) = \frac{1}{\sqrt{2\pi}\sigma y} e^{-\frac{(lny-\mu)^2}{2\sigma^2}}; 0 < y < \infty \tag{3}$$

The probability distribution function for this random variable is

$$F(z) = \frac{1}{\sqrt{2\pi}} \int_0^z e^{-\frac{z^2}{2}} dz\, ; z = \frac{lny-\mu}{\sigma} \tag{4}$$

Parameters of Lognormal Function

The parameters of the distribution are derived in relations to that of normal distribution using the moment generating function and the as unbiased estimator. Hence, we define the function

$$\varphi_y(t) = \int_0^\infty e^{yt} f(y)\, dy \tag{5}$$

The mean and second moments are defined as $\frac{d\varphi_y(0)}{dt} = E(y)$ and $\left.\frac{\partial^2 y}{\partial t^2}\right|_{t=0} = E(y^2)$.

$$\varphi_y(t) = \frac{1}{\sqrt{2\pi}\sigma} \sum_0^\infty \frac{t^n}{n!} \int_0^\infty y^{(n-1)} e^{-\frac{(lny-\mu)^2}{2\sigma^2}} dy \tag{6}$$

This is the moment generating function for lognormal function to be used to relate the mean and variance as in this sequence. The first and second derivatives are:

$$\varphi_y(t) = \frac{1}{\sqrt{2\pi}\sigma} \sum_{n=1}^\infty \frac{t^n}{n!} \int_0^\infty y^n e^{-\frac{(lny-\mu)^2}{2\sigma^2}} dy \tag{7}$$

and

$$\varphi_y(t) = \frac{1}{\sqrt{2\pi}\sigma} \sum_{n=2}^\infty \frac{t^n}{n!} \int_0^\infty y^n e^{-\frac{(lny-\mu)^2}{2\sigma^2}} dy \tag{8}$$

The expectancy is given at $t = 0$ which automatically gives $n = 1$ and using equation 8 for second moment at $t = 0, n = 2$. That is,

$$E(y) = \frac{e^\mu}{\sqrt{2\pi}} \int_0^\infty e^{-\frac{1}{2}(z^2-2\sigma z)} dz = e^{\left(\mu+\frac{\sigma^2}{2}\right)} \tag{9}$$

Also the second moment

$$E(y^2) = \frac{e^{2\mu}}{\sqrt{2\pi}} \int_0^\infty e^{-\frac{1}{2}(z^2-4\sigma z+4\sigma^2-4\sigma^2)} dz = e^{(2\mu+2\sigma)} \tag{10}$$

By the definition of variance as, $\sigma^2 = E(Y^2) - (EY)^2$, then

$$\sigma_y{}^2 = e^{2(\mu+\sigma)} - e^{2\mu+\sigma} = \left(E(Y)\right)^2 \left(e^{\sigma^2} - 1\right) \tag{11}$$

The median is the value at which $F(z) = 0.5$ and this implies $ln(y) = \mu; median = e^\mu$. The mode is the point at which derivative of eqn.3 is zero and it is $lny = \mu - \sigma^2$ and $y = e^{\mu+\frac{\sigma^2}{2}-\frac{3\sigma^2}{2}} = E(Y)e^{-\frac{3\sigma^2}{2}}$, The mode is given as

$$Mode = E(y)e^{-1.5\sigma^2} \tag{12}$$

Equation 4 gives

$$F(Z) = \frac{1}{2}\left(1 + \text{erf}\left(\frac{Z}{\sqrt{2}}\right)\right) \tag{13}$$

Where erf () is the error function and equation 13 is tabulated as Gussian or Normal table in statistical text.

The economic model is based on the acceptance that normal distribution will approximate the outcomes and is

$$C < \frac{D\,(x_A + \sqrt{\sigma}Z)}{(1+w)} \tag{14}$$

In this case z is the number of normal score and is negative and standard deviation is evaluated for a single well outcomes.

Interpretation of Model

We used this method of analysis to evaluate the drilling program for Benton fields whose sizes distribution is in the table 1. The assumption that discoveries will follow lognormal distribution and that we can make a reasonable calculation of rate of return from 10 well-drilling program. The data are arrange in ascending order and the mean and standard deviation are so calculated by acceptance the equally likely chance of a single well will discover any of these fields.

Table 1: Benton field data in barrels

30000	450000	75000	1850000	240000
45000	900000	135000	4600000	19000000

The mean of the samples is 27325000 Bls and standard deviation of 5585820 Bls from which the normal mean and standard deviation are calculated as 13.998 and 1.645 bls. The probability density curves are shown in fig. 1 both skewed and normalized curves. The distribution curve in fig. 2

Discussions

The data were estimated for some number of wells in this geological area. Tables 2 and 3 show the analysis of the data. For group of wells the mean is constant and standard deviation varies as the number increases and further analysis indicated distribution tends toward normal distribution. The distributions of field sizes with number of well are shown in fig.5 and is parametized by the level of confidences.

Conclusions

Statistical decision methods have been developed for economic interpretations of highly uncertain ventures; the method are based on participating in enough ventures to guarantee survival. The method permits expressing character of financial ventures in terms of confidence limit. Lognormal analysis of the available information provides the confidence level.

Making decisions on risk ventures involves more than looking at the average rate of return or even average present worth. The number of risk ventures we can rationally undertake with limited capital will be probabilistically predicted and it depends upon the chance of success of each venture and the possible magnitude of a success.

This method of analysis will be helpful when an investor is concerned about an amount of time and money involve in a series of risk venture to be sure of success. This analysis puts quantitative number on the chance of having various degree of success.

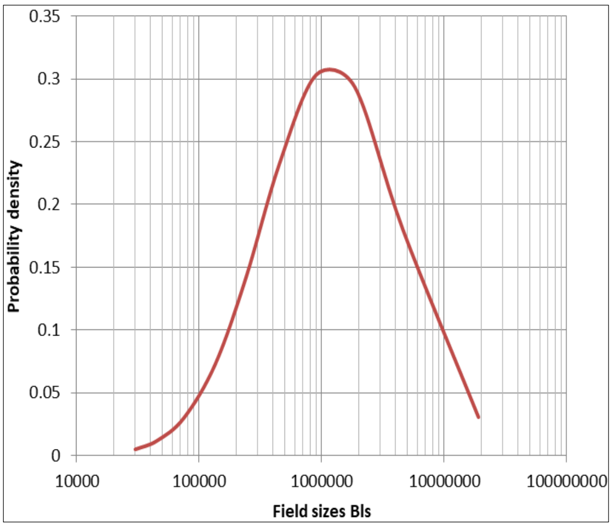

Fig 1: skewed representation plots

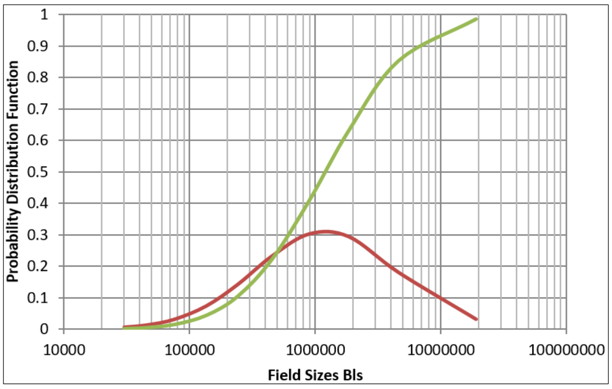

Fig 2: Log-normal probability distribution function

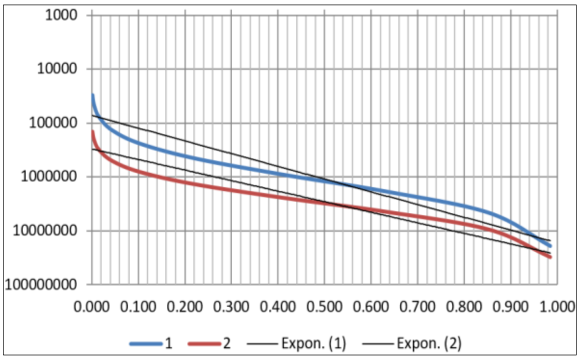

Fig 3: Distribution curves comparing two wells

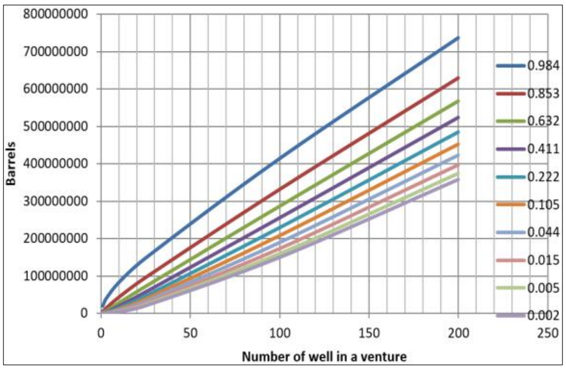

Fig 4: well outcome distribution in relation to expectancy

Table 2: Simulation outputs per number of attempts

Z	Fz	F(Z)	1	2	3	10	25	100	200
2.153375	0.030617	0.984355	19000000	30612115	39607237	81912485	1.47E+08	4.14E+08	7.37E+08
1.047346	0.179757	0.85253	4600000	9456694	14095881	42610384	95447246	3.31E+08	6.29E+08
0.337066	0.293908	0.631966	1850000	4447579	7260275	28005270	72191826	2.87E+08	5.68E+08
-0.2248	0.303326	0.411066	900000	2448853	4295562	20093251	57883247	2.56E+08	5.24E+08
-0.76531	0.232114	0.222045	450000	1379292	2592711	14599686	46802139	2.29E+08	4.85E+08
-1.25548	0.141451	0.104652	240000	819526	1640227	10928277	38598580	2.08E+08	4.52E+08
-1.70414	0.072823	0.044177	135000	508885.1	1078691	8383295	32356836	1.9E+08	4.23E+08
-2.16249	0.030021	0.01529	75000	312758.2	703007.9	6394283	27021335	1.73E+08	3.96E+08
-2.56082	0.011719	0.005221	45000	204870.1	484583.7	5053239	23104381	1.59E+08	3.74E+08
-2.87699	0.004961	0.002007	30000	146434.6	360667.1	4192099	20403740	1.5E+08	3.58E+08

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