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Anisotropic media and well interference analysis

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Abstract
This paper typically studied the pressure transient response for interference test in the anisotropic media in consideration with the stress sensitivity of permeability. Combined with anisotropy characteristics of permeability, second order tensor for directional permeability test method is established by means the linear flow analytical equation in fractured anisotropic media is presented. Based on Darcy's law of fluid flow, tensor theory and coordinate transformation principle, the two-dimensional permeability tensor model for matching type curve with field data are derived for multi-well interpretation analysis. It is extended to the reservoir in situ condition, and the mathematical model of the tensor permeability for three well system in pressure-sensitive anisotropic media is established. The results have a significant impact in understanding the seepage law in the fracture.

Keywords: anisotropic media, interference analysis

Introduction

The variation in permeability in different planes or directions is known as anisotropic permeability. Anisotropic permeability is especially important when dealing with horizontal or partially penetrated wells since flow occurs in both the vertical and horizontal planes. Johnson et al. (1948) [11]; Johnson and Breston (1951) [10] analyzed a series of oil well cores by cutting them into small horizontal plugs and observed that the permeability varies with the direction in which the plug is cut. Compared with the theory of Darcy for isotropy porous media, an extension for anisotropic media has been developed by Ferrandon (1948) [6] upon theoretical grounds, in which permeability is represented as a symmetric tensor. Based on the study of Johnson et al. (1948) [11]; Johnson and Breston (1951) [10] and Ferrandon's theory (1948), Scheidegger (1954) obtained the substantiation of the tensor theory. Liakopoulos (1960, 1965a, b) [13, 15] described the permeability in homogenous anisotropic soils by a second rank, symmetric, positive definite tensor. Chapuis (1989) [3] studied how densification influences the anisotropy of sands and sedimentary rocks by laboratory test method, and found the anisotropy of sandstone increases with densification. Leung (1986) [12] presented the physical and mathematical interpretation for the cross terms in the Cartesian 2D permeability tensor. Marcus (1962) [18] and Parsons (1964) [22] studied the laboratory results of directional permeability, and concluded that the full-diameter core analysis measurements made on rocks from almost any reservoir suggest that permeability is an anisotropic property. Vertical permeability is almost always lower than permeability measured in any horizontal direction. There are usually sound petrographic reasons why this should be so. However, the practice of making measurements on plugs which are of length greater than their diameter magnifies the true anisotropy. Prediction of the exact permeability anisotropy factor to apply in a given reservoir situation, using measurements made in the laboratory, is probably difficult or impossible. The aim of this paper is to present to develop some logical models of anisotropy measurements from multi-well pressure transient data.

The term permeability anisotropy, as used here, may be defined as the ratio or product of directional permeability (k_{max}) to minimum direction horizontal permeability (k_{min}). It is a quantity which affects, directly or indirectly, the solution of many problems arising in petroleum reservoirs. The actual ratio measured on a rock sample depends not only on lithology, but also on the geometric shape of the sample. Welltest is very effective in providing analytical interpretable designed mechanism to estimate areally anisotropy from interference tests conducted from a numbers of well system arrangement.

Model Development

The form of Darcy's law that is most widely used in formulating fluid flow equations in reservoir simulators assumes that the coordinate system is aligned with the principal axes of the permeability tensor. The resulting diagonalized permeability greatly simplifies the fluid flow diffusivity equations. The solutions to the simplified equations are readily written down so that the well test data interpretations would be by type curve matching. We consider anisotropic reservoir as one with differing permeability in the two orthogonal directions in xy-plane. The mathematics of permeability-tensor depends on isomorphic transformation, that is, there must be a transformation matrix such that $L^T M_k L$ is diagonal. For two dimensional horizontal permeability variations shown in fig.1

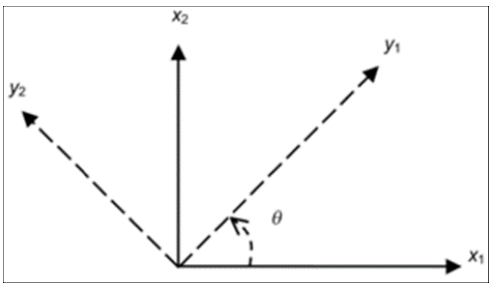

Fig 1: Rotation of the coordinate system.

$$L = \begin{pmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{pmatrix} \quad (1)$$

$$M_k = \begin{pmatrix} K_x & K_{xy} \\ K_{xy} & K_y \end{pmatrix} \quad (2)$$

$\bar{k} = L^T ML$, where $\bar{k} = (k_{max}, k_{min})$

by treating permeability as a dyadic or as a matrix. We are interested here in the matrix representation.

The diagonal permeability elements $\{k_x, k_y\}$ represent the dependence of flow rate in one direction on pressure differences in the same direction. The off-diagonal permeability elements $\{k_{xy}, k_{yx}\}$ account for the dependence of flow rate in one direction on pressure differences in orthogonal directions. Darcy equation for each directional fluid velocity gives the corresponding set of two equations demonstrating this dependence.

$$\begin{matrix} -\mu v_x \\ -\mu v_y \end{matrix} = \begin{pmatrix} k_x & k_{xy} \\ k_{yx} & k_y \end{pmatrix} \begin{matrix} \frac{\partial p}{\partial x} \\ \frac{\partial p}{\partial y} \end{matrix} \quad (3)$$

We can the algebra as

$$L^T M_k L = \begin{pmatrix} K_x cos^2\theta + 2K_{xy} cos\theta sin\theta + K_y sin^2\theta & K_{xy} cos2\theta - \left(\frac{K_x - K_y}{2} sin2\theta\right) \\ K_{xy} cos2\theta - \left(\frac{K_x - K_y}{2} sin2\theta\right) & K_x sin^2\theta - 2K_{yx} cos\theta sin\theta + K_y cos^2\theta \end{pmatrix} \quad (4)$$

and by setting the elements of the lower diagonal to zero such that

$$K_{xy} cos2\theta - \left(\frac{K_x - K_y}{2} sin2\theta\right) = 0 \quad (5)$$

And we have the two principal directions of anisotropy,

$$tan2\theta = \frac{2K_{xy}}{K_x - K_y} \quad (6)$$

These give two values of θ at right angle corresponding to maxima and minima flow directions. The values of permeability in these directions can be designated as principal anisotropic of the region, while we call the directions, the principal axes of the horizontal plane of the porous domain.

$$cos2\theta = \frac{K_x - K_y}{\sqrt{(K_x - K_y)^2 + 4K_{xy}^2}} \quad (7)$$

$$sin2\theta = \frac{2K_{xy}}{\sqrt{(K_x - K_y)^2 + 4K_{xy}^2}} \quad (12)$$

Without loss of generality we take the directional permeabilities are

$$K_{min} = K_x sin^2\theta + 2K_{yx} cos\theta sin\theta + K_y cos^2\theta \quad (9)$$

and

$$K_{min} = K_x sin^2\theta - 2K_{yx} cos\theta sin\theta + K_y cos^2\theta \quad (10)$$

Substituting equations 7 and 8, then eqns. 9 and 10 can be written as

$$k_{max} = \frac{1}{2}\left\{k_x + k_y + \sqrt{(k_x - k_y)^2 + 4k^2_{xy}}\right\} \quad (11)$$

$$k_{max} = \frac{1}{2}\left\{k_x + k_y - \sqrt{(k_x - k_y)^2 + 4k^2_{xy}}\right\} \quad (12)$$

These two directional permeabilities are also the eigenvalues of the permeability matrix and their sum is given as

$$k_{max} + k_{min} = k_x + k_y \quad (13)$$

The product of these permabilities is

$$k_{max}k_{min} = \left(k_x k_y - k_{xy}^2\right) \quad (14)$$

The moment equation of rigidity of porous medium using quadratic function

$$I_{oR} = K_x sin^2\theta - 2K_{yx} cos\theta sin\theta + K_y cos^2\theta \quad (15)$$

And define the transformation variables

$$X = \frac{RCos\theta}{\sqrt{K_{max}k_{min}}} \ ; Y = \frac{R\ Sin\theta}{\sqrt{K_{max}k_{min}}} \quad (16)$$

Which transformed eqn. 15 to radial equation

$$r^2 = \frac{k_x y^2 - 2K_{yx} xy + k_y x^2}{k_{max}k_{min}} \quad (17)$$

Defining the dimensionless variables

$$P_D = \frac{2\pi\sqrt{k_{max}k_{min}}h(p_i - p(r,t))}{q\mu B} \ ; \ t_{RD} = \frac{0.0002637t}{\emptyset\mu c_t R^2}$$

From which the derived solution to diffusivity equation is obviously

$$P_D = \frac{-1}{2} Ei\left(\frac{-r_D^2}{4t_D}\right) \quad (18)$$

In real variable we have

$$\Delta p(r,t) = -\frac{70.6q\mu B}{h\sqrt{k_{max}k_{min}}} Ei\left(-\frac{\emptyset\mu c_t r^2}{4 * 0.0002637t} \frac{(k_x X^2 + k_y Y^2 - 2k_{xy} XY)}{k_{max}k_{min}}\right) \quad (19)$$

This signifies that the region of investigation is ellipse while for isotropic reservoir it is circular.

Permeability variations for fractured anisotropic media

Fractured reservoirs' media consists of matrix and effective fractures. The fracture distribution is complex and variable in these reservoirs with strong characters of anisotropy of reservoirs. This section develops a simple, fast and accurate method to interpret and calculate 2D tensor permeability of fractured anisotropy media. Combining numerical simulation results and experiment results, 2D tensor permeability is derived and the variation mechanism of 2D tensor permeability in fractured anisotropic media has been revealed. According to the polar form of elliptic equation, permeability elliptic is derived. With ellipsoidal permeability, the change law of permeability value in principal direction is studied. 2D permeability tensor model for fractured media is proposed on basis of the co-ordinate transformation principle. Based on the quantitative characterization of 2D tensor permeability and transformation about the principal directions of permeability model, mathematical model a pressure distribution in fractured media is established. The method provides a theoretical basis for the determination of storativity and other parameters in fractured reservoirs, and the results are significant in understanding the fluid flow in fractured anisotropic media.

Introduction

It is well known that fractured porous media is often anisotropic, which means permeability values depend on the direction at which it is measured. The permeability parallel to the fracture is usually greater than the perpendicular direction. Many investigators have attempted to either experimentally develop a procedure to measure directional permeabilities or to develop a mathematical model to calculate anisotropy permeability of a system.

Now the equation of pressure in 2-dimensions is

$$k_x\frac{\partial^2 p}{\partial x^2}+k_y\frac{\partial^2 p}{\partial y^2}=\emptyset\mu c_t\frac{\partial p}{\partial t} \tag{20}$$

Where $(k_x,k_y)=(k_{max},k_{min})$ are the principal permeabilities which are constant along the principal axes (x,y), respectively. With the initial condition,

$$p(x,y,0)=p_i \tag{21}$$

And the inner boundary condition

$$q=\int_0^{2\pi} v_r r dr \tag{22}$$

$$v_r=-\frac{1}{\mu}\left(k_x\frac{\partial p}{\partial x}\cos\theta+k_y\frac{\partial p}{\partial y}\sin\theta\right) \tag{23}$$

Such that by transformation into new variables

$$X=\frac{x}{\sqrt{k_x}}=\frac{r\cos\theta}{\sqrt{k_x}},Y=\frac{y}{\sqrt{k_y}}=\frac{r\sin\theta}{\sqrt{k_y}} \tag{24}$$

The diffusion eqn 2 becomes

$$\frac{\partial^2 p}{\partial X^2}+\frac{\partial^2 p}{\partial Y^2}=\emptyset\mu c_t\frac{\partial p}{\partial t} \tag{25}$$

While we transform the normal velocity, eqn. 22, to the form

$$v_r=-\frac{1}{\mu}\left(\sqrt{k_x}\cos\theta\frac{\partial p}{\partial X}+\sqrt{k_y}\sin\theta\frac{\partial p}{\partial Y}\right) \tag{26}$$

Therefore the distance from origin to the point (X,Y), $R^2=X^2+Y^2=r^2\left(\frac{cos^2\theta}{k_x}+\frac{sin^2\theta}{k_y}\right)$ can be used to write down equations in term of radius as

$$\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial p}{\partial R}\right)=\emptyset\mu c_t\frac{\partial p}{\partial t} \tag{27}$$

And

$$v_r=-\frac{\left(X\sqrt{k_x}\cos\theta+Y\sqrt{k_y}\sin\theta\right)}{\mu R}\frac{\partial p}{\partial R}=\frac{-r}{\mu R}\frac{\partial p}{\partial R} \tag{28}$$

As has already been known, pressure response to radial flow in a homogeneous isotropic formation for production at a constant rate from a well of infinitesimal radius, $r_w=0$, can be described by the E_i function. Therefore the radial equations has the solution

$$p_D(R,t)=\frac{-1}{2}Ei\left(-\frac{\emptyset\mu c_t R^2}{4t}\right) \tag{15}$$

$$p_D(r,t)=\frac{-1}{2}Ei\left(-\frac{\emptyset\mu c_t r^2}{4t}\left(\frac{k_y\cos^2\theta+k_x sin^2\theta}{k_x k_y}\right)\right) \tag{27}$$

$$p_D(r,t)=\frac{p_i-p(r,t)}{q\mu B\sqrt{k_x k_y}} \tag{28}$$

The effective reservoir storativity and transmissibility are defined as

$$S=\emptyset h c_t\left(\sqrt{\frac{k_y}{k_x}}cos^2\theta+\sqrt{\frac{k_x}{k_y}}sin^2\theta\right),T=\frac{h}{\mu}\sqrt{k_x k_y}=\sqrt{T_x T_y} \tag{29}$$

In field units the solution will be written as

$$\Delta p(r,t)=\frac{-70.6q\mu B}{h\sqrt{k_x k_y}}Ei\left(-\frac{Sr^2}{4*0.0002637Tt}\right) \tag{30}$$

Analysis of Results

We have demonstrated that because a pressure response is described by the $Ei-$ function does not necessarily imply that the flow to the well is radial. Therefore, applying analysis methods based upon the conventional interpretation method to $Ei-$ ''look alike'' pressure responses will not necessarily result in the correct reservoir parameter values. Pressure transient data from flow in a homogeneous anisotropic formation is complex to interpret and involve trial and error approach. The applications of equation 19 to multi- well interference data involve the following sequential steps

- Plot the pressure-time data from each of the observation wells on log-log graph sheets.
- Superpose each plot on the type curve of figure 2 (plotted from table 1) and find match points for each
- A single $p_D{:}\,\Delta p$ match-point is picked for all the plots from which the average permeability $\sqrt{k_{max}k_{min}}\frac{h}{\mu}=141.2qB\left(\frac{P_D}{\Delta p}\right)_{match}$
- The $\frac{t_D}{{r^2}_D}-t$, match points are difference for all the wells in anisotropy medium. This generates $(n,3)-$algebraic equations in $\left(k_x,k_y,k_{xy}\right)$. Using the equation

$$\frac{t_D}{{r_D}^2}=\frac{0.0002637k_{max}k_{min}t}{\emptyset\mu c_t\left(k_x x^2+k_y y^2-2k_{xy}xy\right)} \tag{31}$$

$$k_x x^2+k_y y^2-2k_{xy}xy=\frac{0.0002637k_{max}k_{min}}{\emptyset\mu c_t\left(\frac{t_D}{{r^2}_D}/t\right)_{match}} \tag{32}$$

$$\begin{pmatrix} {x_1}^2 & \cdots & x_i y_1 \\ \vdots & \ddots & \vdots \\ {x_n}^2 & \cdots & x_n y_n \end{pmatrix} \begin{pmatrix} \emptyset c_t k_x \\ k_y \emptyset c_t \\ k_{xy} c_t \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \\ \vdots \\ \vdots \\ d_n \end{pmatrix} \quad (33)$$

Solving eqn. 30 numerically is simply in parametric terms for unique solutions $\{\varepsilon_1, \varepsilon_2, \varepsilon_3\}$ such that

$$\begin{matrix} k_x & & \varepsilon_1/\theta c_t \\ k_y & = & \varepsilon_2/\theta c_t \\ k_{xy} & & \varepsilon_3/\theta c_t \end{matrix} \quad (34)$$

By the auxiliary eqn. 14 we calculate $\emptyset c_t -$ product as

$$k_{max}k_{min} = (\varepsilon_1\varepsilon_2 - \varepsilon^2{}_3)/(\theta c_t)^2 \quad (35)$$

And

$$\emptyset c_t = \sqrt{\frac{(\varepsilon_1\varepsilon_2 - \varepsilon^2{}_3)}{k_{max}k_{min}}} \quad (36)$$

Conclusions

A homogeneous but areally anisotropic formation has directional permeability, with maximum and minimum values along two orthogonal axes, called the principal axes, lying in the plane of the formation. Massive faulting, for example, can cause such preferential permeability; the higher-permeability trends parallel the fault alignment. Likewise, braided stream deposits or natural fractures generally show highly directional permeability, with ease of flow being greatest along the braids or fractures.

Where $k_x\ and k_y, k_{xy}$, porosity,$\emptyset$, total compressibility $c_t$porosity, $\emptyset$, total compressibility c_t are the unknown directional permeabilities, while axes x, y the well coordinate-locations, and fluid viscosity μ are all known constant during the test, throughout the formation. Equation (30) implies an areally anisotropic transmissibility, with $T_x = k_x h/\mu\ \ and\ T_y = k_y h/\mu$ as principal values and variable storages,

Thus, a formation with areally anisotropic transmissibility and constant storage, subject to the diffusivity equation of variable coefficients is described by the Ei solution with the apparent reservoir parameters $T = \sqrt{T_x T_y} and\ S$. The test-determined (effective) transmissibility is equal to the geometric mean of the transmissibilities along the major and minor anisotropic axes. The effective transmissibility is thus not a constant value. It depends of the orientation and upon the angle θ, that is, it varies with the orientation of the observation well. Thus, the test determined values of $T\ and\ S$ show the formation to have constant transmissibility and variable storage, whereas the reverse is actually true.

Table 1: Simulated Exponential integral equation

td/rD^2	pD	td/rD^2	pD	td/rD^2	pD	td/rD^2	pD	td/rD^2	pD
0.025	5.13E-08	0.155	0.006081	0.285	0.03357	8	1.459	19790	5.351
0.035	1.69E-06	0.165	0.007438	0.295	0.03644	16	1.799	97910	6.15
0.045	1.32E-05	0.175	0.008935	0.305	0.0394	32	2.141	488500	6.954
0.055	5.26E-05	0.185	0.01057	0.315	0.04245	64	2.486	2442000	7.759
0.065	0.000143	0.195	0.01234	0.325	0.04557	128	2.832	12210000	8.563
0.075	0.00031	0.205	0.01423	0.335	0.04878	256	3.178	61040000	9.368
0.085	0.000573	0.215	0.01626	0.345	0.05205	257.8	3.181	3.05E+08	10.17
0.095	0.00095	0.225	0.0184	0.355	0.0554	258.8	3.183	1.53E+09	10.98
0.105	0.001454	0.235	0.02066	0.365	0.05881	263.8	3.193		
0.115	0.002093	0.245	0.02304	0.5	0.1101	288.8	3.238		
0.125	0.002873	0.255	0.02552	1	0.3301	413.8	3.417		
0.135	0.003797	0.265	0.0281	2	0.7019	1039	3.878		
0.145	0.004867	0.275	0.03079	4	1.108	4164	4.572		

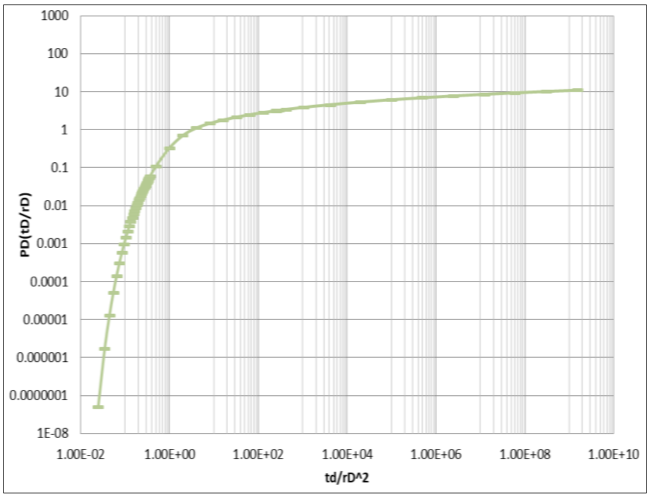

Fig 2: Type-Curve Plot of $p_D\ vs. \frac{t_D}{{r_D}^2}$

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