

Hall Effect on MHD flow past over moving inclined plate with mass transfer

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Abstract

In this paper, we examine the effect of Hall current on unsteady MHD flow past a moving inclined plate with mass diffusion. The fluid considered is viscous, electrically conducting, incompressible. The concentration level of the fluid near the plate increase linearly with time. The Laplace transform technique has been used to find the solutions for the velocity profile. The velocity profile have been studied for different parameters like angle of inclination of plate, Schmidt number, Hall parameter, magnetic field parameter, Mass Grashof number and time. The effects of parameters are shown graphically for different parameters. The results are found to be in good agreement and the data obtained is in concurrence with the actual flow phenomenon.

Keywords: MHD flow, Mass transfer, Hall current

1. Introduction

The Study of MHD flow with heat transfer in the presence of Hall current has important applications in many branches of engineering and applied sciences. Hall effect on unsteady hydromagnetic flow was analyzed by Prasada *et al.* [5]. Ram *et al.* [6] have worked on Hall effect on heat and mass transfer flow through porous medium. Hall effect on magnetohydrodynamic boundary layer flow over a continuous moving plat plate was investigated by Watanabe and Pop [10]. Effect of Hall current and heat transfer on the flow in a porous medium with slip condition was considered by Hayat and Abbas [4]. Deka [3] has examined Hall effect on MHD flow past an accelerated plate. Further, Deka along with Sahoo [7] have explained Hall effect on hydromagnetic flow past an accelerated horizontal porous plate. Heat and mass transfer in elastic viscous fluid past an impulsively started infinite vertical plate with Hall effects was developed by Chaudhary and Kumar [2]. Syamala *et al.* [9] have discussed effect of Hall current on MHD flow of a couple stress fluid through a porous medium in a parallel plate channel in presence of effect of inclined magnetic field. Alam *et al.* [1] have considered heat and mass transfer in MHD free convection flow over an inclined plate with Hall current. Unsteady flow through porous media past on moving vertical plate with variable temperature in the presence of inclined magnetic field was studied by us along with Kumar and Singh [8]. In this present paper we have investigated effects of Hall effect on unsteady MHD flow past over moving inclined plate with mass transfer. The velocity profile has been observed with the help of graphs.

2. Mathematical formulation

The geometrical model of the problem is shown in Figure-1

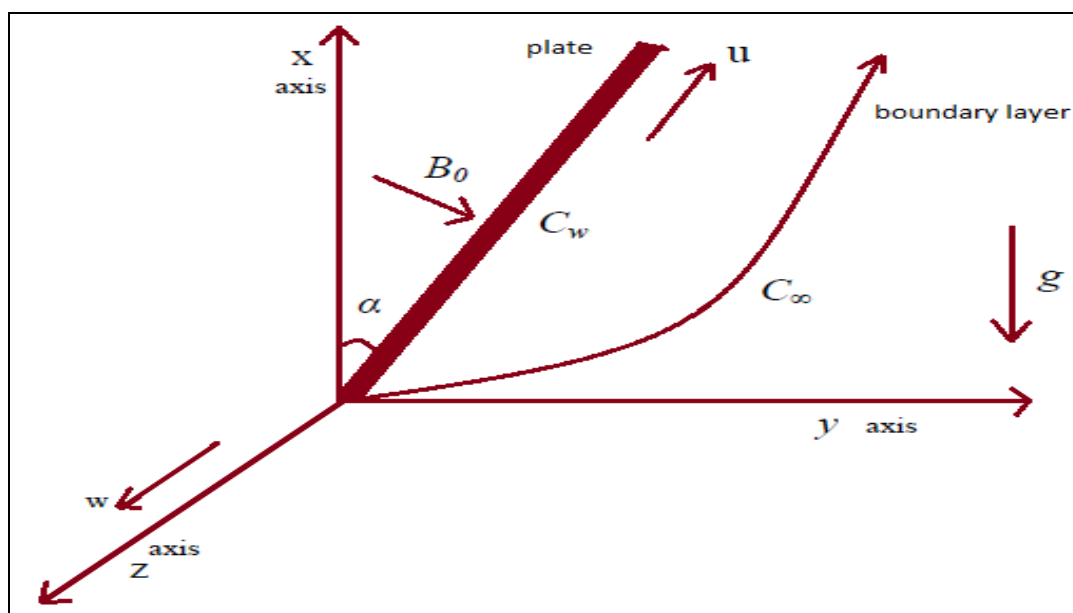


Fig 1: Physical model

Considered an unsteady viscous, incompressible and electrically conducting fluid past moving inclined plate with velocity u_0 . The plate is electrically non conducting. The x axis is taken along the vertical plane and y axis is normal to it. Thus the y axis lies in the horizontal plane. The plate is inclined at angle α from vertical. A uniform magnetic field B is assumed to be applied in the y-direction. Initially the fluid and plate are at the same concentration C_∞ in the stationary condition. A time $t > 0$, the concentration level near the plate is raised linearly with respect to time. Due to the Hall effects there will be two components of the momentum equation, which are as under.

The flow modal is as under:

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta \text{Cos}\phi (C - C_\infty) - \frac{\sigma \mu^2 B_0^2}{\rho(1+m^2)} (u + mw) \tag{1}$$

$$\frac{\partial w}{\partial t} = \nu \frac{\partial^2 w}{\partial y^2} - \frac{\sigma \mu B_0^2}{\rho(1+m^2)} (w - mu) \tag{2}$$

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial y^2} \tag{3}$$

The following boundary conditions have been assumed:

$$\left. \begin{aligned} t \leq 0 : u = 0, w = 0, C = C_\infty \text{ for all the value of } y \\ t > 0 : u = u_0, w = 0, C = C_\infty + (C_w - C_\infty) \frac{u_0^2 t}{\nu} \text{ at } y = 0 \\ u \rightarrow 0, w \rightarrow 0, C \rightarrow C_\infty \text{ as } y \rightarrow \infty \end{aligned} \right\} \tag{4}$$

where u is the velocity of the fluid in x- direction, w - the velocity of the fluid in z- direction, m is the Hall parameter, g - acceleration due to gravity, β - volumetric coefficient of concentration expansion, t - time, C_∞ -is the concentration in the fluid far away from the plate, C is species concentration in the fluid, D is the mass diffusion ν is the kinematic viscosity, ρ is the fluid density and σ electrically conductivity, K is the permeability of the medium, μ is the magnetic permeability. Here $m = \omega_e \tau_e$ with ω_e - cyclotron frequency of electrons and τ_e - electron collision time.

To write the equations (1) - (3) in dimensionless from, we introduce the following non - dimensional quantities:

$$\left. \begin{aligned} \bar{u} = \frac{u}{u_0}, \bar{w} = \frac{w}{u_0}, \bar{y} = \frac{yu_0}{\nu}, Sc = \frac{\nu}{D}, \bar{t} = \frac{tu_0^2}{\nu}, Gm = \frac{g\beta\nu(C_w - C_\infty)}{u_0^3}, C = \frac{C - C_\infty}{C_w - C_\infty} \end{aligned} \right\} \tag{5}$$

Here the symbols used are:

\bar{u} - dimensionless velocity, \bar{w} - dimensionless velocity, M - magnet field parameter, \bar{y} - dimensionless coordinate axis normal to the plate, Sc - Schmidt number, Gm - mass Grashof number.

The dimensionless forms of Equation (1), (2), and (3) are as follows

$$\frac{\partial \bar{u}}{\partial \bar{t}} = \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} + Gm\bar{C} \text{Cos}\phi - \frac{M}{1+m^2} (\bar{u} + m\bar{w}) \tag{6}$$

$$\frac{\partial \bar{w}}{\partial \bar{t}} = \frac{\partial^2 \bar{w}}{\partial \bar{y}^2} - \frac{M}{1+m^2} (\bar{w} - m\bar{u}) \tag{7}$$

$$\frac{\partial \bar{C}}{\partial \bar{t}} = \frac{1}{Sc} \frac{\partial^2 \bar{C}}{\partial \bar{y}^2} \tag{8}$$

The corresponding boundary conditions:

$$\left. \begin{aligned} \bar{t} \leq 0, \bar{u} \leq 0, \bar{C} = 0, \bar{w} = 0, \text{ for all value of } \bar{y} \\ \bar{t} > 0, \bar{u} = 1, \bar{w} = 0, \bar{C} = \bar{t} \text{ at } \bar{y} = 0 \\ \bar{u} \rightarrow 0, \bar{C} \rightarrow 0, \bar{w} \rightarrow 0 \text{ as } \bar{y} \rightarrow 0 \end{aligned} \right\} \quad (9)$$

Dropping the bars and combining the equations (6) and (7), we get

$$\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial y^2} - \frac{M}{1+m^2} q(1-mi) + GmC \cos \phi \quad (10)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial y^2} \quad (11)$$

The Final boundary conditions become:

$$\left. \begin{aligned} t \leq 0, q = 0, C = 0, w = 0 \text{ for all value of } y \\ t > 0, q = 1, C = t, w = 0 \text{ at } y = 0 \\ q \rightarrow 0, C \rightarrow 0, w \rightarrow 0 \text{ as } y \rightarrow \infty \end{aligned} \right\} \quad (12)$$

Here $q = u + iw$, with corresponding boundary conditions

$$q = \frac{1}{2} e^{-\sqrt{a}y} (1 + \operatorname{Erfc}[\frac{2\sqrt{at}-y}{2\sqrt{t}}] + e^{2\sqrt{a}y} \operatorname{Erfc}[\frac{2\sqrt{at}+y}{2\sqrt{t}}]) + \frac{1}{4a^2} y Gm \cos \phi \{A_1(1-a) + \sqrt{a} e^{-\sqrt{a}y} (1 - e^{2\sqrt{a}y} + A_5 + e^{2\sqrt{a}y} A_6) + A_2(1-Sc) - ASc\} + \frac{1}{2a^2 \sqrt{\pi}} y Gm \cos \phi \{a(2e^{-\frac{y^2 Pr}{4t}} \sqrt{t + \sqrt{\pi}y}) (-1 + \operatorname{Erfc}[\frac{y\sqrt{Sc}}{2\sqrt{t}}]) \sqrt{Sc}\} + A_3 (\frac{1}{y} e^{\frac{at}{-1+Sc} - y\sqrt{\frac{a}{-1+Sc}} \sqrt{Sc}} \sqrt{Sc} - \frac{1}{y\sqrt{Sc}} e^{\frac{at}{-1+Sc} - y\sqrt{\frac{a}{-1+Sc}} \sqrt{Sc}}) (-1 + a + Sc). A_4 \} \sqrt{Sc}.$$

$$C = t \{ (1 + \frac{y^2 Sc}{2t}) \operatorname{erfc}[\frac{\sqrt{Sc}}{2\sqrt{t}}] - \frac{y\sqrt{Sc}}{\sqrt{\pi}\sqrt{t}} e^{-\frac{y^2}{4t} Sc} \}.$$

3. Discussion and Results

The numerical value of velocity and skin friction are computed for different parameters like, angle of inclination of plate ϕ , thermal mass Grashof number Gm , magnetic field parameter M , hall parameter m , Schmidt number Sc and time t . The value of main parameters considered are mass Grashof number $Gm=10, 20, 30$ taking others parameters values ($\phi = 30^\circ, M = 2, Pr = 2.01, m = 1, t = 0.2$), angle of inclination of plate $\phi = 15^\circ, 45^\circ, 60^\circ$ with taking others parameters values ($M = 2, Sc = 2.01, Gm = 10, m = 1, t = 0.2$), the magnetic parameter $M= 1, 3, 5$ with taking others parameters values ($\phi = 30^\circ, Sc = 2.01, Gm = 10, m = 1, t = 0.2$), Hall parameter $0.5, 2, 5$ with taking others parameters values ($\phi = 30^\circ, M = 2, Sc = 2.01, Gm = 10, t = 0.2$), Schmidt number $Sc = 2.01, 3, 4$ with taking others parameters values ($\phi = 30^\circ, M = 2, Gm = 10, m = 1, t = 0.2$) and $t=0.4, 0.5$ and 0.6 with taking others parameters values ($\phi = 30^\circ, M = 2, Sc = 2.01, Gm = 10, m = 1$). Figure (4) and (7) shows that primary velocity increased when Gm and t is increased. Figure (2), (3), (5) and (6) shows that primary velocity decreased when ϕ, m, M and Sc is increased. From figures (9), (11), (12) and (13) shows that the secondary velocity increased when $m, M, Sc,$ and t are increased. Figure (8) and (10) shows that secondary velocity decreased when $\phi,$ and Gm is increased

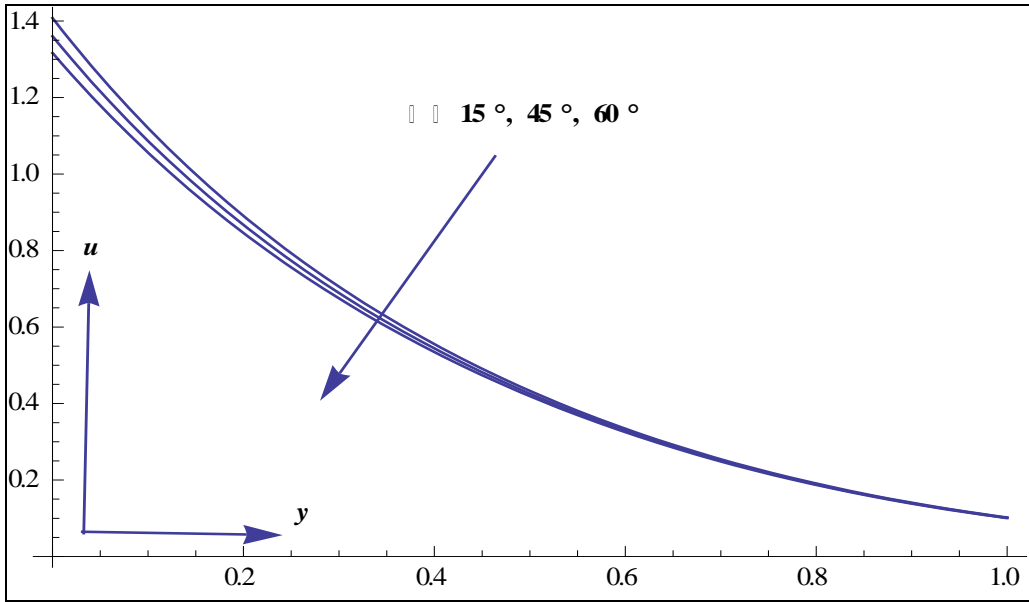


Fig 2: Velocity profiles u for different values of a

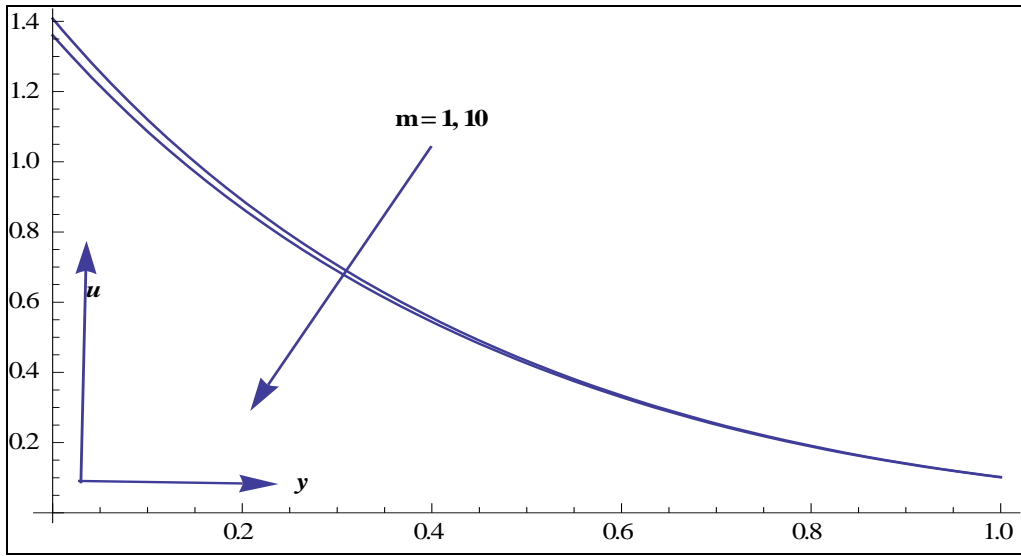


Fig 3: Velocity profiles u for different values of m

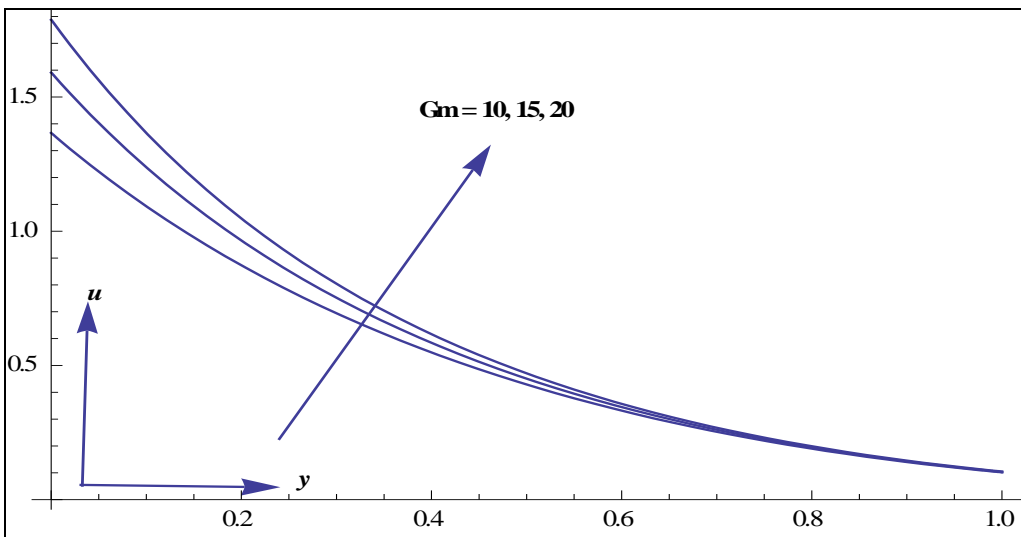


Fig 4: Velocity profiles u for different values of Gm

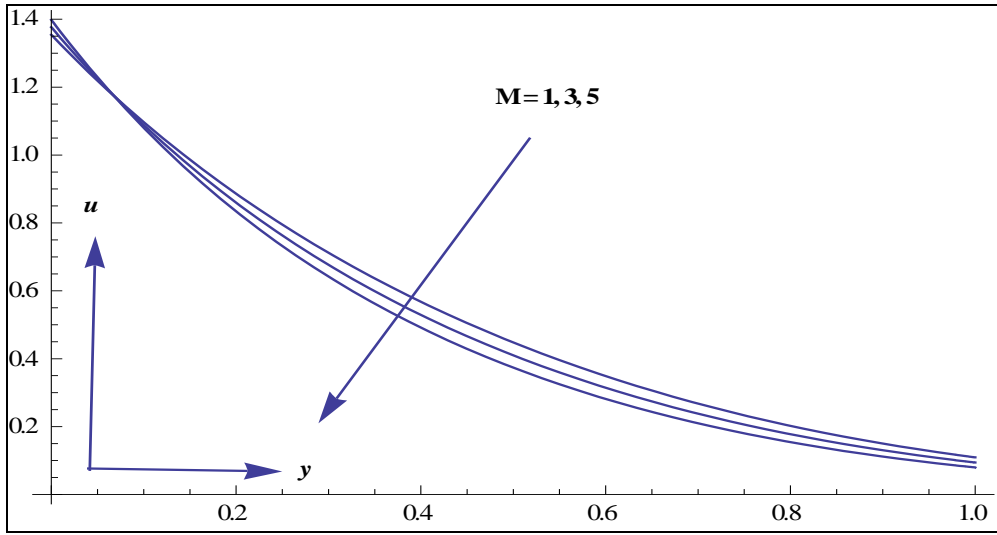


Fig 5: Velocity profiles u for different values of M

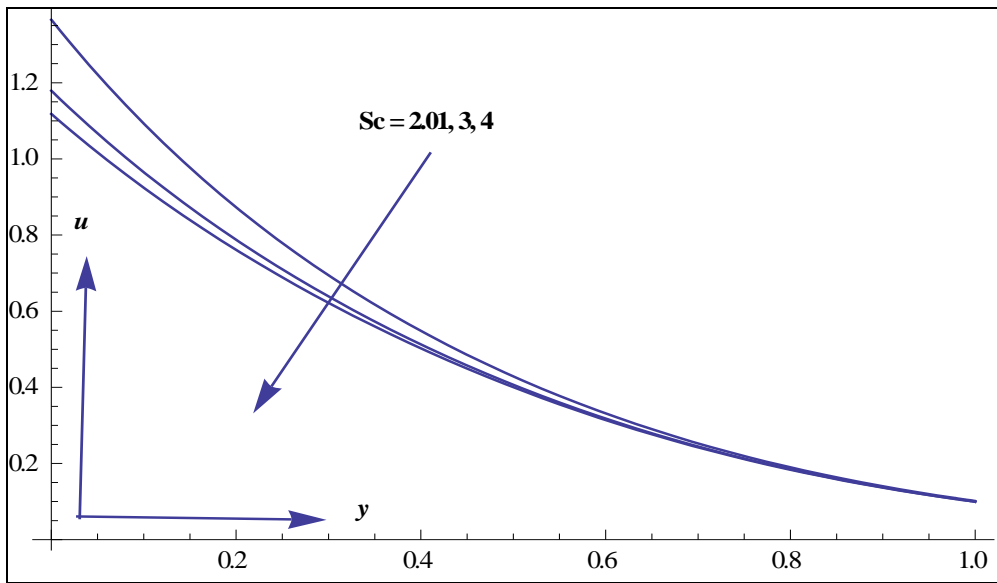


Fig 6: Velocity profiles u for different values of Sc

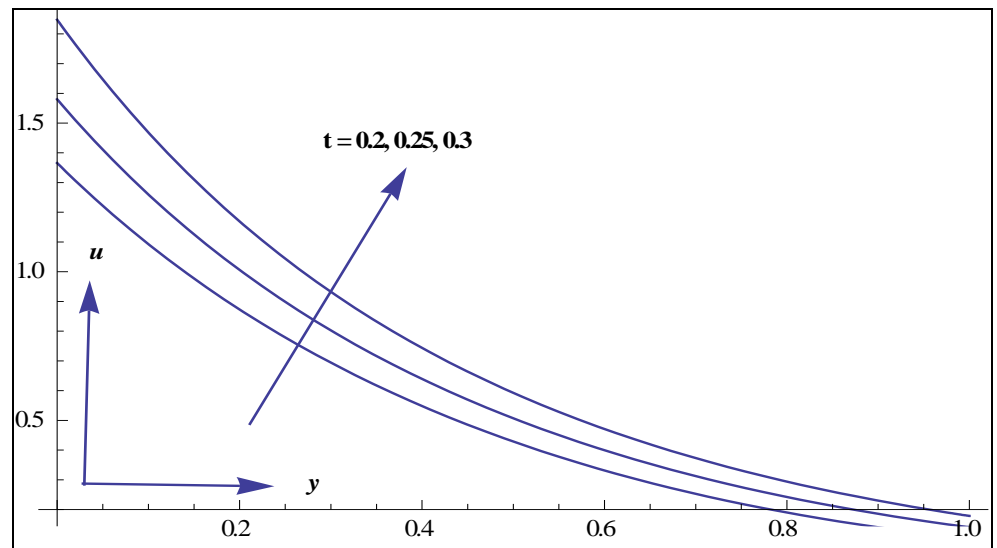


Fig 7: Velocity profile u for different values of t .

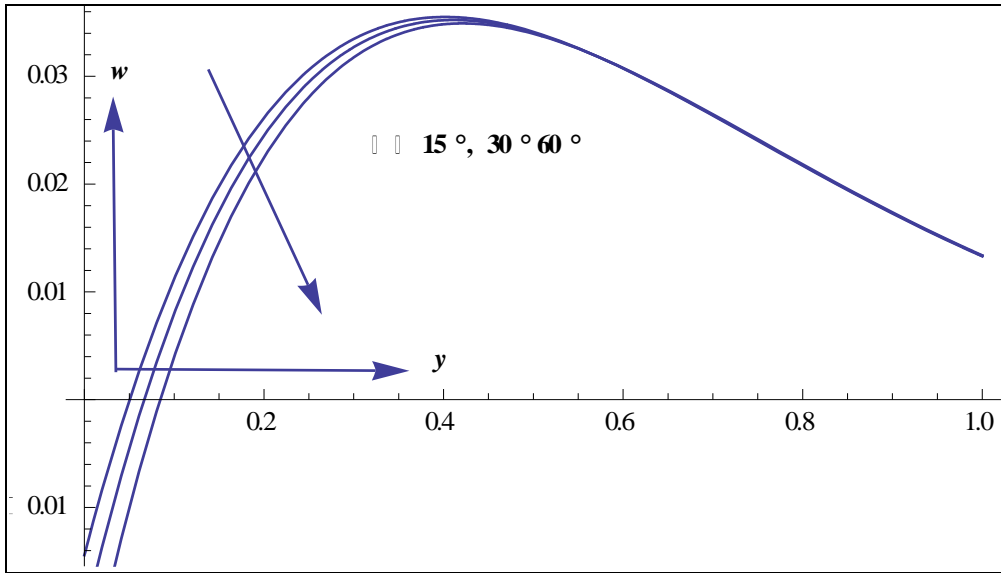


Fig 8: Velocity profiles u for different values of α

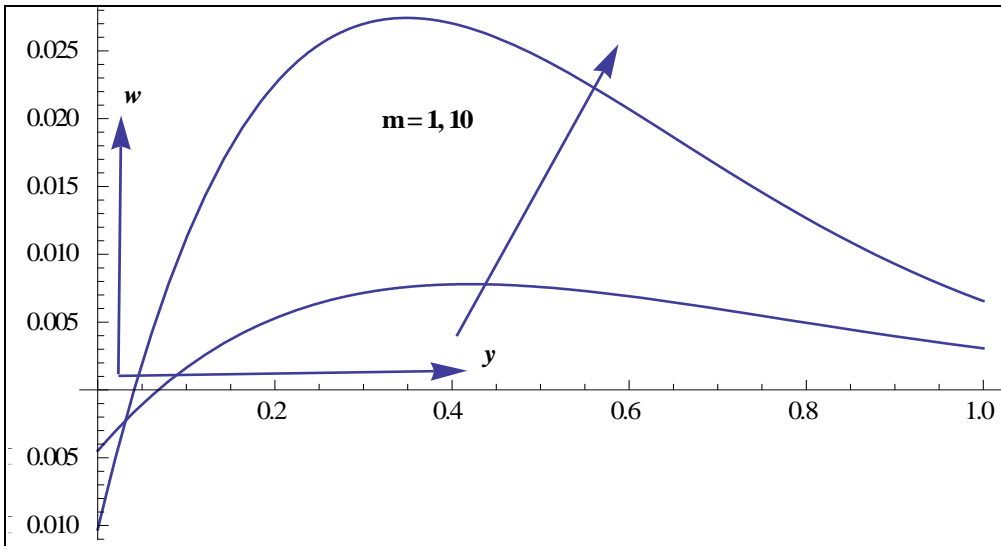


Fig 9: Velocity profile w for different values of m

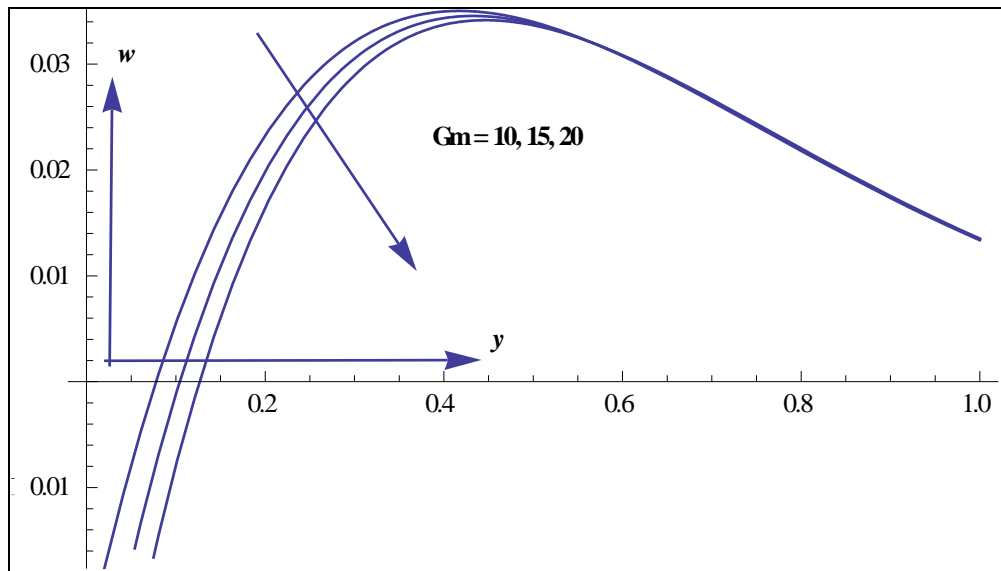


Fig 10: Velocity profile w for different values of Gm

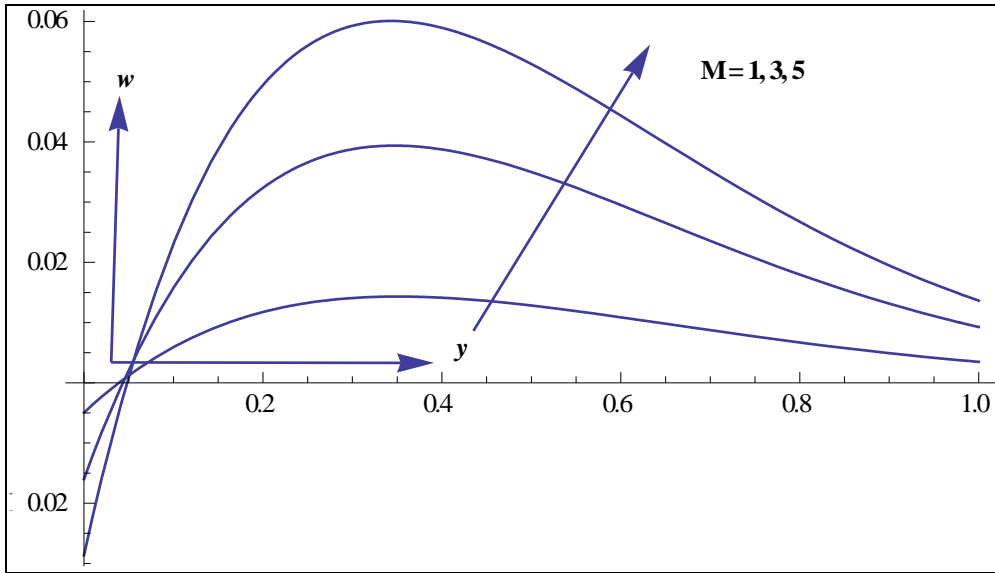


Fig 11: Velocity profile w for different values of M

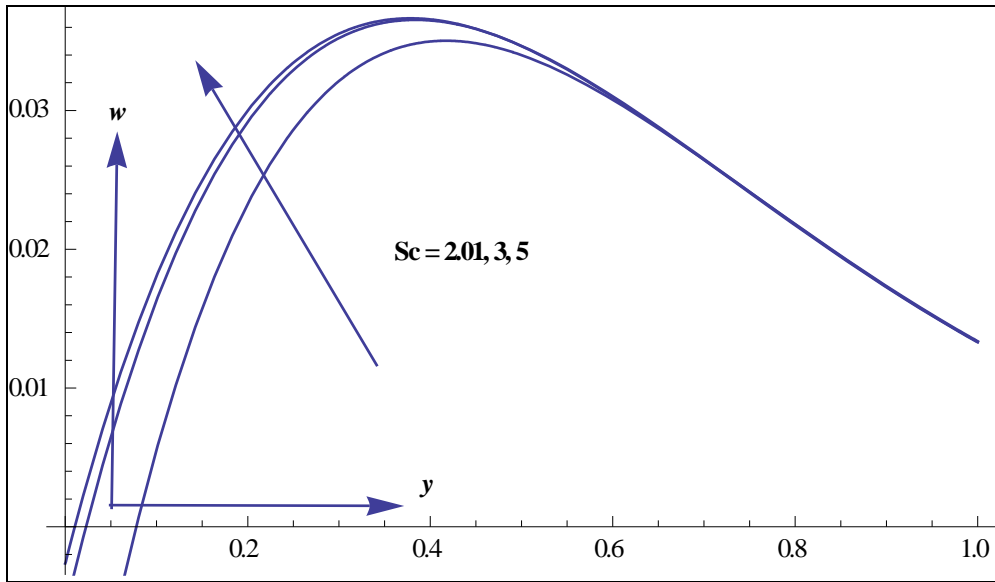


Fig 12: Velocity profile w for different values of Sc .

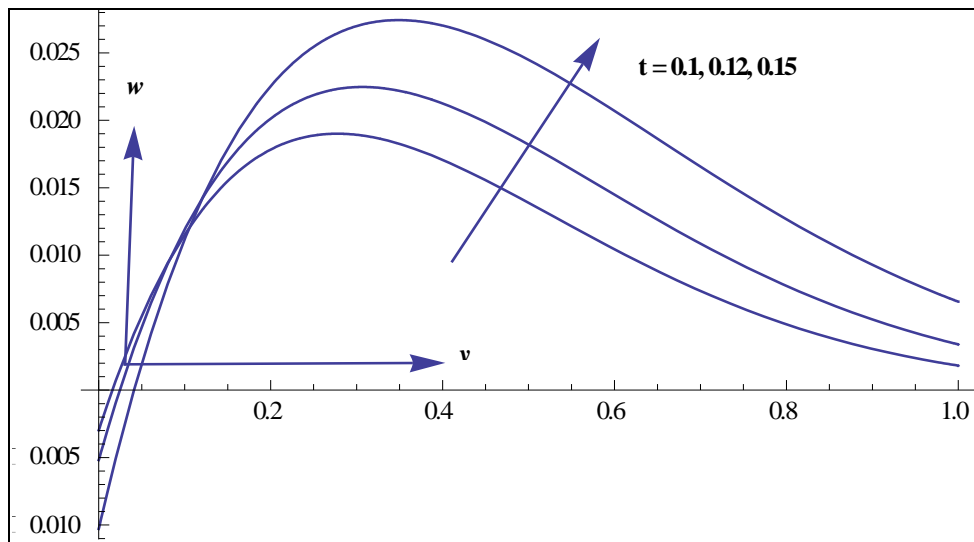


Fig 13: Velocity profile w for different values of t

4. Conclusion

In this paper a theoretical analysis has been done to study the effect of Hall current on Hall Effect on MHD flow past over moving inclined plate with mass transfer and solutions for the model have been derived by using Laplace -transform technique. Some conclusions of the study are as below:

- Primary velocity increases with the increase in mass Grashof number and time.
- Primary velocity decreases with the increase in angle of inclination of the plate, magnetic field parameter, Hall parameter, Schmidt number.
- Secondary velocity increases with increase in time, Hall parameter, Schmidt number and magnetic field parameter.
- Secondary velocity decreases with the increase in angle of inclination of the plate, mass Grashof number.

5. Appendix

$$A_1 = \frac{1}{y} (2e^{-\sqrt{a}y} (1 + e^{2\sqrt{a}y} + A_5 - e^{2\sqrt{a}y} A_6)),$$

$$A_2 = \frac{1}{y} 2A_{11} (-1 - A_{10} + A_8 + A_{10}A_9),$$

$$A_3 = \sqrt{\pi} (A_{10}A_{13} - 1 - A_{10} - A_{12}),$$

$$A_8 = \text{Erf} \left[\frac{1}{2\sqrt{t}} (y - 2t \sqrt{\frac{aSc}{-1+Sc}}) \right],$$

$$A_9 = \text{Erf} \left[\frac{1}{2\sqrt{t}} (y + 2t \sqrt{\frac{aSc}{-1+Sc}}) \right],$$

$$A_{10} = \exp \left(2y \sqrt{\frac{aSc}{-1+Sc}} \right),$$

$$A_{11} = e^{-\frac{at}{-1+Sc}} - y \sqrt{\frac{aSc}{-1+Sc}},$$

$$A_{12} = \text{Erf} \left[\frac{2t \sqrt{\frac{a}{-1+Sc}} - y\sqrt{Sc}}{2t} \right],$$

$$A_{13} = \text{Erf} \left[\frac{2t \sqrt{\frac{a}{-1+Sc}} + y\sqrt{Sc}}{2t} \right],$$

$$A_4 = \frac{2\sqrt{\pi} (-1 + \text{Erf} [\frac{y\sqrt{Sc}}{2\sqrt{t}}])}{y\sqrt{Sc}},$$

$$A_5 = \text{Erf} \left[\frac{2\sqrt{at} - y}{2\sqrt{t}} \right], \quad A_6 = \text{Erf} \left[\frac{2\sqrt{at} + y}{2\sqrt{t}} \right],$$

$$A_7 = \exp\left(\frac{at}{-1+Sc} - y\sqrt{\frac{a}{-1+Sc}}\sqrt{Sc}\right),$$

$$\eta = \frac{y}{2\sqrt{t}}, \quad a = \frac{M(1-im)}{1+m^2}.$$

6. References

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