



A robust production management system under uncertainty environment based on quantum information interchange

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Abstract

In the current complex and dynamic environment, the production environment has undergone fundamental changes. First, in order to cope with the ever-changing environmental challenges, it is of great value to establish a robust production system environment. Second, this paper introduces a measure method of the uncertainty in production system. Third, a kind of quantum information interchange is put forward based on the mode of production, the measure method of uncertainty is applied between the communication protocol in order to ensure the stability of uncertain production system. Four, the uncertainty of environment of production management system has been overcome to maintain robust performance and sustained competitive advantage.

Keywords: production system, uncertain environment, robust operation, quantum information

1. Introduction

1.1 Related Work

The impact of uncertain environment on production system is always a hot topic in international research. Ni (2015) exhibited preventive maintenance opportunities for large production systems Li (2015) [1]. Presented data-driven analysis of downtime impacts in parallel production systems Wu (2016) [2]. Featured production control in a complex production system using approximate dynamic programming Wang (2015) [3]. Researched robust replenishment and production control policy for a single-stage production/inventory system with inventory inaccuracy Shaaban (2016) [4]. Sculpted successful applications and new challenges part 2-sc m, logistics, planning & strategy, scheduling and location [5].

Different researchers provided different to solve the uncertain problem in production system. Jiao (2015) considered an interest rate model in uncertain environment Wen (2015) [6]. Painted the capacitated facility location-allocation problem under uncertain environment Gao (2017) [7]. Sculpted special issue on computational optimization and intelligence in uncertain environment Salmani (2015) [8]. Discussed a bi-objective MIP model for facility layout problem in uncertain environment Szyrowski (2015) [9]. Unveiled Subsea cable tracking in an uncertain environment using particle filters [10].

Many researchers focused on robust operation to undermine the uncertain impact on production system. Tian(2017) made a model a new adaboost.ir soft sensor method for robust operation optimization of ladle furnace refining Zhong (2017) [11]. Built UDE-based robust droop control of inverters in parallel operation Kong (2017) [12]. Etched robust real-time optimization for blending operation of alumina production Zhang (2017) [13]. Draw robust operation of microgrids via two-stage coordinated energy storage and direct load control [14].

The appearance of quantum information provided us a new tool to solve the problem. Zhang (2017) adopted featured a robust flywheel energy storage system discharge strategy for wide speed range operation Diamanti (2015) [15]. Applied distributing secret keys with quantum continuous variables: principle, security and implementations Morton (2015) [16]. Sketched quantum information spin memories in for the long haul Andersen (2015) [17]. Developed furusawa, akira. hybrid discrete- and continuous- variable quantum information Zheng (2015) [18]. Carved probing berezinskii-kosterlitz-thouless phase transition of spin-half XXz chain by quantum fisher information Brandao (2015) [19] offered Strelchuk, Sergii. Quantum conditional mutual information, reconstructed states, and state redistribution Bagarello (2017) [20]. Modelled a model of adaptive decision-making from representation of information environment by quantum fields Frerot (2016) [21]. Depicted a measure of quantum coherence and quantum correlations for many-body systems [22].

1.2 Organization of the Article

Some preliminaries are started in Section 2. In Section 3, a brief retrospect of traditional game are given, using a symbol that fits the one recommended for quantum game, which are studied in Section 4. In Section 4.1, several equivalent definitions of quantum game are given by us; in Section 4.2, the quantities k , k^z , and k^{ne} are introduced and the consistent inequalities are proved in Theorem 1.1; likewise, the rest of quantities k^* , k^{ne} , and k^{sq} and the conclusion of the proof of Theorem 1.1 are recommended in section 4.2. Throughout the full Section 4, example computations are given to demonstrate some of the natures of the family of

games (G_m) alleged in Theorem 1.2. Section 5 involves excess constructions: a topical equivalence between quantum game and rank-one quantum games [Cooney *et al.* 2011] is demonstrated in Section 5.1; in Section 5.2, the part of Theorem 1.2 that was not constituted yet in the examples from Section 4 are proved; in Section 5.3, the family (ξ_m) are recommended, proving Theorem 1.3. Section 6 involves brief interpretations linking the consequences to criterion, but significant, consequences throughout Grothendieck’s inequality in the theory of operator spaces. Section 7 indicates directions for future work. The following symbol are used throughout the whole article.

2. Production System in changeable surrounding

2.1 Changeable matrix in Production system

Outline of the Section. This section begins with the introduction of changeable relations and the establishment of some symbol (Section 2.1). Then, the metric changeable relations are defined in Section 2.2. In Section 2.3, the presence of strong metric uncertainty relations are proved. Specific constructions are given in Section 2.4.

For a positive integer n , $[m] = \{0, \dots, m - 1\}$, are defined.

Another conducive measure of closeness between distributions is the fidelity $\zeta(\mu, \nu) = \sum_{\alpha \in S} \sqrt{u(\alpha)v(\alpha)}$, also called the Bhattacharyya distance and connected to the Hellinger distance. The connection between the fidelity and the trace distance as follow:

$$1 - \zeta(\mu, \nu) \leq \Delta(\mu, \nu) \leq \sqrt{1 - \zeta(\mu, \nu)^2}. \tag{1}$$

The Shannon entropy of a distribution p on X is ruled as $\xi(\mu) = -\sum_{\alpha \in S} \mu(\alpha) \log \mu(\alpha)$ where the log is adopted here and around the article to be base two. Also, the $\xi(\alpha)$ for $\xi(\mu\alpha)$ will be written. The corporate feedback between two random variables β and ε is ruled as $\rho(\beta; \varepsilon) = \xi(\beta) + \xi(\varepsilon) - \xi(\beta, \varepsilon)$. The min-entropy of a distribution μ is ruled as $\xi_{\min}(\mu) = -\log \max_{\alpha} \mu(\alpha)$. It means that a random variable β is a k -source if $\xi_{\min}(\beta) \geq s$. To consult how does the vector composed, except when r has a subscript, the R_j is written. Under the circumstances $r(j)$ used. The binary vector r ’s Hamming weight (number of ones) is bespoke by and the two binary vectors r ’s Hamming distance, r' (number of units that are diverse) is written as $du(r, r')$.

Performing a measurement is the most integrated way to obtain classical information from a quantum state. Described by a positive operator-valued measure (POVM), a measurement which is a set of positive semidefinite operators that sub total the identity. If the density operator displays the state of the quantum system, the potential of observing the feedback labeled i is for all. For a state, the distribution of the results of the measurement of in the basis is bespoke by. can be used. Likewise, for a mixed state, is defined.

The can define the trace distance between density operators working on X . the defines the von Neumann entropy of a quantum state. It will also be bespoke. The quantum mutual information is towards a bipartite state. Fannes, inequality [Fannes 1973], or more accurately an amelioration by Audenaert [2007] is used, which states that for any states and on X ,

$$|\delta(w) - \delta(h)| \leq \Delta(w, h) \log d_X + \zeta_2(\Delta(w, h)), \tag{1}$$

With. $\zeta_2(c) = -c \log(c) - (1-c) \log(1-c)$

2.2 Interchange of Uncertainty

Rather than asking for the conclusion of the measurement on the computational built on the whole space to be uniform, it only requires that the conclusion of a measurement of the X system in its computational basis be close to uniform. Or more precisely, $x \in [d_X]$ is defined,

$$\mu B_s | P \rangle \langle x | = \sum_{y=0}^{d_Y-1} | x \rangle \langle x |^X \langle y |^Y B_s | P \rangle |^2.$$

A metric indetermination connection can be defined. Spontaneously, the larger the X system, the stronger the indetermination connection for a fixed Y system.

Definition 2.1 (*Metric indetermination connection*) Let X and Y be Hilbert spaces. It means that a set $\{B_0, \dots, B_{\sigma-1}\}$ of unified

transformations on XY fulfills an e-metric indetermination connection on X if for all conditions $|P\rangle \in XY$,

$$\frac{1}{O} \sum_{s=0}^{o-1} \Delta \left(\mu_{B_s}^X | P \rangle, \text{unif}([d_X]) \right) \leq c, \tag{3}$$

Remark. Respect that this means that (5) also apply to mixed conditions: for any $P \in E(X \otimes Y)$, $\frac{1}{O} \sum_{s=0}^{o-1} \Delta \left(\mu_{B_s P B_s^o}^X | P \rangle, \text{unif}([d_X]) \right) \leq c$.

Metric indetermination connection Means Entropic indetermination connection. In the next assertion, the condition that a metric indetermination connection bring about an entropic indetermination connection will be shown. It is worthwhile to emphasize that there are no constraints on measurements.

Proposition 2.2. Let $c \in (0,1)$ and $\{B_0, \dots, B_{o-1}\}$ be a series of unitaries on XY fulfilling an e-metric indetermination connection on A :

$$\frac{1}{O} \sum_{s=0}^{o-1} \Delta \left(\mu_{B_s}^X | P \rangle, \text{unif}([d_X]) \right) \leq c, \tag{4}$$

Then

$$\frac{1}{O} \sum_{s=0}^{o-1} \xi(\mu_{B_s}^X | P) \geq (1-c) \log d_X - \zeta_2(c)$$

In which ζ_2 is the binary entropy function.

Explicit link to Low-Distortion Embeddings. Even if the link doesn't been used to low-distortion embeddings particularly, the connection is described as it may have another applications. In the definition of metric indetermination connection, the trace distance is used to compute the distance between allocation. When the closeness of distributions using the fidelity is gauged, it shows that the connection to low- distortion metric embeddings is more clearer. It means

$$\zeta \left(\mu_{B_s}^X | P \rangle, \text{unif}([d_X]) \right) = \frac{1}{\sqrt{d_X}} \sum_{x=0}^{d_X-1} \sqrt{\mu_{B_s}^X | P \rangle(x)} = \frac{1}{\sqrt{d_X}} \sum_{x=0}^{d_X-1} \sqrt{\sum_{y=0}^{d_Y-1} |x\rangle^X \langle y|^Y B_s | P \rangle^2} = \frac{1}{\sqrt{d_X}} \| B_s | P \rangle \|_{\ell_1^X(\ell_2^Y)}$$

In which the norm $\ell_1^X(\ell_2^Y)$ is defined by

Definition 2.3 ($\ell_1^X(\ell_2^Y)$ norm). For a condition $|P\rangle = \sum_{x \in [d_X], y \in [d_Y]} a_{x,y} |x\rangle^X |y\rangle^Y$,

$$\| |P\rangle \|_{\ell_1^X(\ell_2^Y)} = \sum_{x \in [d_X]} \| \{a_{x,y}\}_y \|_2 = \sum_{x \in [d_X]} \sqrt{\sum_{y \in [d_Y]} |a_{x,y}|^2}$$

When the organizations X and Y are distinct from the context, $\| \bullet \|_{12} := \| \bullet \|_{\ell_1^X(\ell_2^Y)}$ is used.

Respect that the norm's definition ups to the computational basis's choice. With reference to the computational bases, the $\ell_1^X(\ell_2^Y)$ norm will always be taken. For $\{B_0, \dots, B_{o-1}\}$ to satisfy an indetermination connection, it needs

$$\frac{1}{O} \sum_s \frac{1}{\sqrt{d_X}} \| B_s | P \rangle \|_{\ell_1^X(\ell_2^Y)} \geq 1 - c, \tag{5}$$

By inducting a new register K that holds the index k , this utterance can be rewritten. All $|P\rangle$ is got by writing $Z = XY$

$$\left\| \frac{1}{\sqrt{O}} \sum_s B_s |P\rangle^z |O\rangle^o \right\|_{\ell_1^{X_1}(\ell_2^Y)} \geq (1-c) \sqrt{O \cdot d_X}, \tag{6}$$

Using the Cauchy-Schwarz inequality, that in this surrounding reads $\|O\rangle\|_{\ell_1^X(\ell_2^Y)} \leq \sqrt{d_X} \|O\rangle\|_2$ for any $|O\rangle \in XY$, it needs that for all $|P\rangle$,

$$\left\| \frac{1}{\sqrt{O}} \sum_s B_s |P\rangle^z |s\rangle^s \right\|_{\ell_1^{X_1}(\ell_2^Y)} \leq \sqrt{O \cdot d_X} \left\| \frac{1}{\sqrt{O}} \sum_o B_o |P\rangle^z |s\rangle^s \right\|_2 = \sqrt{O \cdot d_X}, \tag{7}$$

Rewriting (6) and (7) as

$$(1-c) \leq \frac{1}{\sqrt{O \cdot d_X}} \cdot \frac{\left\| \frac{1}{\sqrt{O}} \sum_s B_s |P\rangle^z |s\rangle^s \right\|_{\ell_1^{X_1}(\ell_2^Y)}}{\left\| \frac{1}{\sqrt{O}} \sum_s B_s |P\rangle^z |s\rangle^s \right\|_2} \leq 1$$

It shows that the vision of C by the linear map $|P\rangle \mapsto \frac{1}{\sqrt{O}} \sum_s B_s |P\rangle \otimes |s\rangle$ is an virtually Euclidean subspace of $(X \otimes \lambda \otimes Y, \ell_1^{X\lambda}(\ell_2^Y))$. In another words, it is an rearing of (Z, ℓ_2) into $(X\lambda Y, \ell_1^{X\lambda}(\ell_2^Y))$ with distortion $1 - (1-c)$ [Matousek 2002] as the map is an isometry (in the sense),

A Comment on the Composition of Metric indetermination connection. Establishing an indetermination connection for a Hilbert space from indetermination connection on smaller Hilbert spaces is a natural way. This composition property is significant for the cryptographic applications of metric indetermination connection lied in the latter half of this article, where setting it swear for the parallel composition's safety of locking-based encryption.

Proposition 2.4. Think about Hilbert spaces X_1, X_2, Y_1, Y_2 . As to $j \in \{0,1\}$, take $\{B_{s_j}^{(j)}\}_{s_j \in [o_j]}$ be a set of unified conversion of $X_j \otimes Y_j$ fulfilling an e-metric indetermination connection on X_j . Then, $\{B_{s_1}^{(1)} \otimes B_{s_2}^{(2)}\}_{s_1, s_2 \in [o_1] \times [o_2]}$ meets $x_2 \in$ -metric indetermination connection on $X_1 \otimes X_2$.

Proof. Make $|P\rangle \in (X_1 \otimes Y_1) \otimes (X_2 \otimes Y_2)$ and make μ_{s_1, s_2} bespeak the distribution procured by taking off $B_{s_1}^{(1)} \otimes B_{s_2}^{(2)} |P$ in the calculating basis of $X_1 \otimes X_2$. The objective of us is to show that

$$\frac{1}{O_1 O_2} \sum_{o_1 \in [o_1], o_2 \in [o_2]} \Delta(\mu_{s_1, s_2}, \text{unif}([d_{X_1}] \times [d_{X_2}])) \leq 2c, \tag{8}$$

As follow,

$$\begin{aligned} \Delta(\mu_{s_1, s_2}, \text{unif}([d_{X_1}] \times [d_{X_2}])) &= \frac{1}{2} \sum_{x_1, x_2} \left| \mu_{s_1, s_2}(x_1, x_2) - \frac{1}{d_{X_1} d_{X_2}} \right| \\ &\leq \frac{1}{2} \sum_{x_1, x_2} \left| \mu_{s_1, s_2}(x_1, x_2) - \frac{\mu^{X_1}_{s_1, s_2}(x_1)}{d_{X_2}} \right| + \frac{1}{2} \sum_{x_1, x_2} \left| \frac{\mu^{X_1}_{s_1, s_2}(x_1)}{d_{X_2}} - \frac{1}{d_{X_1} d_{X_2}} \right| \\ &= \frac{1}{2} \sum_{x_1} \mu^{X_1}_{s_1, s_2}(x_1) \sum_{x_2} \left| \frac{\mu_{s_1, s_2}(x_1, x_2)}{\mu^{X_1}_{s_1, s_2}(x_1)} - d_{X_2} \right| + \frac{1}{2} \sum_{x_2} \left| \mu^{X_1}_{s_1, s_2}(x_1) - \frac{1}{d_{X_1}} \right| \end{aligned}$$

In which $\mu^{X_1}_{s_1, s_2}(x_1) := \sum_{s_2} \mu_{s_1, s_2}(x_1, x_2)$ is the conclusion distribution of taking off the A1 system of

$\mathbf{B}_{s_1}^{(1)} \otimes \mathbf{B}_{s_2}^{(2)} | P \rangle$. The distribution $\mu_{s_1, s_2}^{x_1}$ can also be served as the conclusion of taking off the hybrid state $\mathbf{B}_{s_1}^{(1)} P^{X_1 Y_1} \mathbf{B}_{s_1}^{(1)}$ in the calculating foundation $\{ |x_1\rangle \}$. Therefore, for any $s_2 \in [o_2]$, $\frac{1}{o_1} \sum_{s_1} \Delta(\mu_{s_1, s_2}^{x_1}, \text{unif}([d_{x_1}])) \leq 2c$

Furthermore, for $x_1 \in [d_{x_1}]$, the distribution on $[d_{x_1}]$ defined by $\frac{\mu_{s_1, s_2}(x_1, x_2)}{\mu_{s_1, s_2}(x_1)}$ is the conclusion distribution of estimating in the calculating basis of X_2 the condition.

3. Rough manipulation analysis with changeable information

Respect that the random variable $\|Q\|_{12}^{XY}$ is distributed as the $\ell_1^{d_x}(\ell_2^{2d_y})$ norm of a *real* random vector picked from the rotation invariant dimension on the boundary $E^{2d_x d_y - 1}$. The integers n is defined and m the norm $\ell_1^m(\ell_2^n)$ of a real m n-dimensional vector $\{r_{j,i}\}_{j \in [m], i \in [n]}$ as for the sophisticated situation (Definition 2.3)

$$\|r\|_{\ell_1^m(\ell_2^n)} = \sum_j \sqrt{\sum_i |r_{j,i}|^2}, \tag{9}$$

It be note worthy that only the dimension of the systems is prescribed as the systems themselves are not concerned here. In other confirmation, $\|\cdot\|_{12}$ is used as a shorthand for $\|\cdot\|_{\ell_1^{d_x}(\ell_2^{2d_y})}$.

The purpose of us is to identify the expected value $\psi\{\|\Lambda\|_{12}\}$ in which Λ has circulation invariant distribution on the practical sphere E^{e-1} and $e = 2d$ with $d = d_x d_y$. As to this, it begins with the concerning $\psi\{\|D\|_{12}\}$ and $\{\psi\{\|\Lambda\|_{12}\}\}$ in which D has a normal Gaussian distribution on V^q . By changing to polar coordinates, it can be seen

$$\psi\{\|\Lambda\|_{12}\} = \int_{V^q} \|\alpha\|_{12} \frac{e^{-\frac{1}{2}\sum_{j=1}^q \alpha_j^2}}{(2\pi)^{q/2}} d\alpha = \int_0^\infty \int_{E^{q-1}} \|v\theta\|_{12} \frac{e^{-v^2/2}}{(2\pi)^{q/2}} \cdot \frac{q\pi^{q/2} dh(\theta)}{U(\frac{q}{2} + 1)} v^{q-1}, \tag{10}$$

Now, it can be calculated

$$\psi\{\|D\|_{12}\} = \int_{V^E} \|\alpha\|_{12} \frac{e^{-\frac{1}{2}\|\alpha\|_{12}^2}}{(2\pi)^{q/2}} d\alpha = \sum_{j=1}^{d_x} \int_{V^E} \|\alpha_j\|_{12} \frac{e^{-\frac{1}{2}\|\alpha_j\|_{12}^2}}{(2\pi)^{q/2}} d\alpha, \tag{11}$$

In which $\alpha = (\alpha_1, \dots, \alpha_{d_x})$ where $x_i \in R^{2d_b}$ is decomposed. As all the terms of the sum are equal

$$\psi\{\|D\|_{12}\} = \int_{V^{2d_y}} \|\alpha_0\|_{12} \frac{e^{-\frac{1}{2}\|\alpha_0\|_{12}^2}}{(2\pi)^{d_y}} d\alpha_0 \left(\int_{V^{2d_y}} \|\alpha_0\|_{12} \frac{e^{-\frac{1}{2}\|\alpha_1\|_{12}^2}}{(2\pi)^{d_y}} d\alpha_1 \right)^{d_x - 1} = d_x \frac{\sqrt{2}U(\frac{2d_y + 1}{2})}{U(d_y)} \int_{E^{2d_y - 1}} \|\theta\|_{12} dh(\theta)$$

In order to obtain the second equality, the similar reasoning is used as for equation (36). Eq is concluded using (36)

$$\psi\{\|Q\|_{\ell_1^x(\ell_2^y)}\} = \psi\{\|\Lambda\|_{12}\} = d_x \frac{U(d_y + \frac{1}{2})}{U(d_y)} \frac{U(d_x d_y)}{U(d_x d_y + \frac{1}{2})}, \tag{12}$$

Now, the inequality is proved in the lemmas' statement. The two facts about the T function is used as follow: $\log G$ is convex and

for all $\omega > 0$, $U(\omega + 1) = \omega U(\omega)$. Using Holder's inequality, the first property can be seen for instance, and the second using integration by parts. Using these properties, we have

$$U(\alpha + \frac{1}{2}) \leq \frac{1}{2} \log U(\alpha) + \frac{1}{2} U(\alpha + 1) = \frac{1}{2} \log(\alpha U(\alpha)^2) = \log(\sqrt{\alpha} U(\alpha)^2)$$

Now two more normal results are stated that can be invested in stead of Lemma 2.7. Note that this is not the standard version of Levy's lemma which uses the median instead of the expectation.

Lemma A.1 (Levy's Lemma). *Make $f: Z^d \rightarrow V$ and $u > 0$ be so that in spite of pure states $|Q_1\rangle, |Q_2\rangle$ in Z^d*

$$|\varphi(|Q_1\rangle) - \varphi(|Q_2\rangle)| \leq u \| |Q_1\rangle - |Q_2\rangle \|_2$$

Make $|Q\rangle$ be a arbitrary pure state in measurement d. Therefore, as to all $0 < p$,

$$\delta\{|\varphi(|Q\rangle) - \psi\{\varphi(Q)\}| \geq p\} \leq 4 \exp(-\frac{p^2 d}{zu^2})$$

Where c is a constant. $z = 18\pi^2$ can be taken.

PROOF. The concentration of a Lipschitz function on the real sphere E^{2d-1} can be instead studied. It be note worthy that the induced function (that also called u) is still n-Lipschitz.

we get

$$\delta\{|\varphi(|Q\rangle) - \psi\{\varphi(Q)\}| \geq p\} \leq 2 \exp(-\frac{p^2 (2d)}{18\pi^2 u^2}) + 2 \exp(-\frac{2}{2\pi^2}) \leq 4 \exp(-\frac{p^2 d}{9\pi^2 u^2})$$

The average of specialty extraneous variable having sub-Gaussian tails is well centralized around its prospects is proved in the following lemma. The proofuses criterion methods for proving concentration inequalities.

Lemma A.2. Let $x, y \geq 1$, and t a positive integer. Expect β is a extraneous variable with zero mean, fulfilling the tail departs for all $u > 0$

$$\delta\{\beta \geq u\} \leq x e^{-yu^2} \text{ and } \delta\{\beta \leq -u\} \leq x e^{-yu^2}.$$

Make β_1, \dots, β_t be independent copies of β . Accordingly, suppose $p > 0$ and $p^2 y > 16x^2 \pi$,

$$\delta\left\{\frac{1}{t} \sum_{s=1}^t \beta_s \geq p\right\} \leq \exp(-\frac{p^2 y t}{2})$$

Proof. As for any $\lambda > 0$, employing Markov's inequality

$$p\left\{\sum_{s=1}^t \beta_s \geq \lambda\right\} = \delta\left\{\exp\left(\lambda \sum_{s=1}^t \beta_s\right) \geq \exp(\lambda t p)\right\} \leq \psi\left\{\exp\left(\lambda \sum_{s=1}^t \beta_s\right)\right\} e^{-\lambda t p} = \psi\{e^{l\beta}\}^t e^{-l\lambda p}$$

Now the moment generating function $\psi\{e^{l\beta}\}$ is bounded of β using the tail bounds.

$$\begin{aligned} \psi\{e^{l\beta}\} &= \int_0^\infty \delta\{e^{l\beta} \geq b\} db = \int_0^\infty \psi\left\{\beta \geq \frac{\ln b}{l}\right\} db = \int_0^1 \psi\left\{\beta \geq \frac{\ln b}{l}\right\} db + \int_1^\infty \psi\left\{\beta \geq \frac{\ln b}{l}\right\} db \\ &\leq 1 + \int_0^\infty x \exp(-\frac{y \ln^2 b}{l^2}) db = 1 + x \int_0^\infty \exp(-\frac{y \omega^2}{l^2}) e^\omega d\omega \end{aligned}$$

by making the change of variable $\omega = \log b$.

$$\begin{aligned} \psi\{e^{l\beta}\} &\leq 1+x \int_0^\infty \exp(-\frac{y}{l^2}(\omega-\frac{l^2}{2y})^2 + \frac{l^2}{4y})d\omega \leq 1+x \exp(\frac{l^2}{4y}) \int_{-\infty}^\infty \exp(-\frac{y}{l^2}(\omega-\frac{l^2}{2y})^2)d\omega \\ &= 1+x \exp(\frac{l^2}{4y}) \frac{l}{\sqrt{2y}} \int_{-\infty}^\infty \exp(-\frac{b^2}{2})db = 1+x \frac{\sqrt{2\pi}l}{\sqrt{2y}} \exp(\frac{l^2}{4y}) \leq 2 \max(1, a \frac{\sqrt{\pi}l}{\sqrt{y}}, \exp(\frac{l^2}{4y})) \end{aligned}$$

$l = 2py$ is chosen (although it is not the first-rank option, it makes expressions more simple),

$$\begin{aligned} \delta\{\sum_{s=1}^o \beta_s \geq op\} &\leq \max(2^o, (2x \frac{\sqrt{\pi}l}{\sqrt{y}}) \cdot \exp(\frac{l^2 o}{4y})) \exp(-lop) \\ &= \max(\exp(-2p^2 yo + o \ln 2), \exp(p^2 yo - 2p^2 yo + t \ln(4x\sqrt{\pi}p\sqrt{y}))) \\ &= \max\{\exp((-2p^2 y + \ln 2)o), \exp((-p^2 yo + \ln(4x\sqrt{\pi}p\sqrt{y}))o)\} \end{aligned}$$

Claim. As to all $z \geq 1$ and $\alpha \geq z$
 $2\ln(z\alpha) - \alpha \leq -\alpha^2$

The function $\alpha \mapsto \frac{\alpha}{2} - \frac{1}{2} \ln(z\alpha)$ is adding to $\alpha \geq 1$. It is enough to reveal that it is nonnegative for $\alpha = z$. In order to see that,

the function of $\beta \mapsto \beta - \frac{1}{2} \ln(\beta^2)$ is differentiated to demonstrate that for all $\sigma \geq 1$, $\sigma - \ln \sigma^2 \geq 0$ can be got. This reveals the claim.

In the use of this inequality, it shows $p^2 y \geq 16x^2 \pi$, $p^2 y + \ln(4x\sqrt{\pi}p\sqrt{y}) \leq -\frac{p^2 y}{2}$ and $-2p^2 y + \ln 2 \leq -\frac{p^2 y}{2}$

Finally, $\delta\{\sum_{s=1}^o \beta_s \geq op\} \leq \exp(-\frac{p^2 y}{2})$

4. Example analysis

Let $\{B_0, \dots, B_{o-1}\}$ be a set of unitary transformations of $X \otimes Y \cong Z$ and define

$$|w\rangle^{XYZ\lambda'} = \frac{1}{d_X d_Y} \sum_{s \in \{o\}, x \in [d_X], y \in [d_Y]} |x\rangle^X |y\rangle^Y (B_s^o |x\rangle \otimes |y\rangle)^Z |x\rangle^{X'} |s\rangle^{\lambda'}$$

If $\{B_0, \dots, B_{o-1}\}$ meets an e-metric unsure relation, accordingly, a locking effect is got employing Theorem 3.3 and Proposition 3.2. In reality, it shows that $\rho^{\leftarrow}(X; Z\lambda) \leq \log d_Z$ and $\rho^{\leftarrow}(X; Z)_w \leq c \log d_X + \zeta_2(c)$. Therefore, employing (29),

$\psi_\varphi(X; X'Y\lambda)_w = \xi(X)_w - \rho^{\leftarrow}(X; Z)_w \geq (1-c) \log d_X - \zeta_2(c)$ can be got.

and discarding the system λ' of dimension o a separable state $\psi_\varphi(X; X'Y)_\rho = 0$ is obtained

This appendix's objective is to give an example of a construction that is not a locking scheme to elucidate what is needed to gain a locking scheme. The 2×2 Pauli matrices are the four matrices $\{\Pi, \mathcal{G}_\alpha, \mathcal{G}_\omega, j\mathcal{G}_\alpha \mathcal{G}_\beta\}$ in which

$$h_\alpha = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ and } h_\omega = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

For bit strings $b, r \in \{0,1\}^m$, the unitary operation $\sigma_\alpha^b \sigma_\omega^r$ is defined on $(\mathbb{Z}^2)^{\otimes m}$ by

$$h_\alpha^b h_\omega^r = h_\alpha^{b_1} h_\omega^{r_1} \otimes \dots \otimes h_\alpha^{b_m} h_\omega^{r_m}.$$

It was shown in Ambainis *et al.* [2000] that one can encrypt an n -qubits state $|P\rangle$ perfectly employing a key (B, R) of $2n$ bits. To encrypt $|P\rangle$, one simply applies $h_\alpha^B h_\omega^R$ to $|P\rangle$.

This can be regarded as a quantum version of one-time pad encryption. By all means, this encryption scheme also defines a $(0, 0)$ -locking scheme, but the size of the key is $2m$ bits. Come back that the assumption need to be used that the message is random to reduce the key size to $O(\text{polylog}(m))$ bits.

Proposition D.1. Consider an c -locking scheme c of the form $(c(\alpha, s) = (b, r)) = h_\alpha^b h_\omega^r |\alpha\rangle$ in which the message $\alpha \in \{0,1\}^m$ and the key $b, r \in \{0,1\}^m$ (see Definition 3.1). Expect that the secret key λ is picked steady from a set $E \subseteq \{0,1\}^{2m}$. Accordingly $|E| \geq (1-c)2^m$.

PROOF. Let X be the message (X is uniformly distributed over $\{0,1\}^m$) and (B, R) be the key. The key is steady distributed on S . It shows that a measurement in the computational basis gives a lot of information about β . Let ρ be the outcome of measuring $c(\beta, \lambda)$ in the computational basis. For $\alpha, j \in \{0,1\}^m$,

$$\delta\{\beta = \alpha | \rho = j\} = \delta\{\rho = j | \beta = \alpha\} = \frac{1}{|E|} \sum_{(b,r) \in E} \left| \langle j | h_\alpha^b h_\omega^r | \alpha \rangle \right|^2$$

Respecting that the term $\left| \langle j | h_\alpha^b h_\omega^r | \alpha \rangle \right|^2 \in \{0,1\}$, for any fixed j , there are at most $|E|$ different values of x for which $\delta\{\beta = \alpha | \rho = j\} > 0$. Therefore, defining $G = \{\alpha \in \{0,1\}^m : \delta\{\beta = \alpha | \rho = j\} = 0\}$,

$$\Delta(\mu\beta | [\rho = j], \mu\beta) \geq \delta\{\beta \in G\} - P\{\beta \in G | \rho = j\} = \frac{G}{2^m} = 1 - \frac{|E|}{2^m} \text{ can be obtained}$$

By the definition of a locking scheme, it shows

$$\Delta(\mu\alpha_{[\rho=j]}, \mu\alpha) \leq c$$

Which concludes the proof.

5. Conclusion

In this article, unsure relationships is studied on the basis of a relationship with low- distortion embeddings of (\mathbb{Z}^d, ℓ_2) into (\mathbb{Z}^d, ℓ_1) . The intuition behind this relationship is very easy. Expect the measurements defined by a set of orthonormal bases $\{Y_0, Y_1, \dots, Y_{o-1}\}$ of $(\mathbb{Z}^2)^{\otimes m}$. The bases $\{Y_0, Y_1, \dots, Y_{o-1}\}$ meet an unsure connection if for every n -qubit state $|P\rangle$ and “most” bases B_k , the vector standing $|P\rangle$ in Y_s is “extend”. One way to quantify the extend of a vector is by its ℓ_1 norm, that is, the sum of the absolute values of its parts. A vector $|P\rangle \in (\mathbb{Z}^2)^{\otimes m}$ of unit ℓ_2 norm is well spread if its ℓ_1 norm is close to its maximal value of $\sqrt{2^m}$.

It is concluded by employing the metric unsure relations of Theorems 2.5 and 2.16. For the distinct construction, it is still need to be argued that the encoding can be calculated by a quantum circuit of size $O(m^2 \text{ polylog}(m/c))$ and depth $O(m \text{ polylog}(m/c))$ employing classical precomputation. It be note worthy that the unitaries from Theorem 2.16 need to be applied in superposition. The only thing need to be precomputed is an irreducible polynomial of degree m over $\zeta_2[\beta]$. Accordingly, employing the same argument as in the proof of Lemma 2.10, the unitary operation can be calculated that takes as input the state $|i\rangle$ and outcomes the

state $R_i|P\rangle$ employing a circuit of size $O(m^2 \text{ polylog } m)$ and depth $O(m \text{ polylog } m)$. As the permutation extractor employed can be carried out by a quantum circuit of size $O(m \text{ polylog}(m^c))$, the unitary transformation $|s\rangle \otimes |P\rangle \mapsto |s\rangle \otimes B_s|P\rangle$ can be calculated by a quantum circuit of size $O(m^2 \text{ polylog}(m^c))$ and depth $O(m \text{ polylog}(m^c))$.

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7. References

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