International Journal of Advanced Engineering and Technology
ISSN: 2456-7655
Impact Factor: RJIF 5.54
www.newengineeringjournal.com
Volume 1; Issue 5; November 2017; Page No. 27-34

A quantum game model in china haze governance

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Abstract
Here a quantum game is introduced in China haze governance. First, the problem China haze governance is modeled as a three-party game including the government, enterprise, and citizen of players with quantum states. Second, it is shown that the quantum game is more suitable for China haze governance problem in the behavior analysis different from for classic game. Third, the characteristic of quantum entanglement takes great advantage over the classic game with no entanglement. Fore, a numerical analysis is used to prove proposed quantum game in China Haze Governance, it is shown that entangled quantum state gives an advantage over classic game.

Keywords: china haze, environmental governance, quamtum game, quantum entangle

1. Introduction
1.1 Related Work
Now China haze problem became a global problem. Chang Wenyuan (2017) painted the association of weather patterns with haze episodes: Recognition by PM2.5 oriented circulation classification applied in Xiamen, Southeastern China [1].Zhang, Ji(2017) llustrated chemical composition, source, and process of urban aerosols during winter haze formation in Northeast China [2]. Xu, Jianming (2017) put forward Role of climate anomalies on decadal variation in the occurrence of wintertime haze in the Yangtze River Delta, China [3]. Ma Siqi (2017) gave characteristics and cause analysis of heavy haze in Changchun City in Northeast China [4]. Lu, Miaomiao (2017) displayed source tagging modeling study of heavy haze episodes under complex regional transport processes over Wuhan megacity, Central China [5]. Liu, Lei (2017) implied Paying more attention on keeping eye health in dust-haze weather in China [6]. Liang, Linlin (2017) made a research on chemical characteristics of PM2.5 lduring summer at a background site of the Yangtze River Delta in China [7].
Many researchers provided all kinds of methods to solve the haze problem. Jin, Rong (2017) concerned gas-particle phase partitioning and particle size distribution of chlorinated and brominated polycyclic aromatic hydrocarbons in haze [8]. Sulong, Nor Azura (2017) discussed source apportionment and health risk assessment among specific age groups during haze and non-haze episodes in Kuala Lumpur, Malaysia [9]. Ding, Dong-Sheng (2015) sculpted quantum Storage of Orbital Angular Momentum Entanglement in an Atomic Ensemble [10]. Saglamyurek, Erhan (2015) promoted quantum storage of entangled telecom-wavelength photons in an erbium-doped optical fibre [11]. Hosten, Onur (2016) framed measurement noise 100 times lower than the quantum-projection limit using entangled atoms [12]. Kogias, Ioannis (2015) extended quantification of Gaussian Quantum Steering [13]. Li, Tongcang (2016) modelled quantum superposition, entanglement, and state teleportation of a microorganism on an electromechanical oscillator [14]. Pichler, Hannes (2015) depicted quantum optics of chiral spin networks [15]. Benson, David (2015) implied water Governance in a Comparative Perspective: From IWRM to a 'Nexus' Approach [16].
Quantum mechanics provided us a helpful tool to seek the answer of complex systems. Calabrese, Pasquale (2016) unveiled quantum quenches in 1+1 dimensional conformal field theories [17]. Cvitanovic, C (2015) introduced improving knowledge exchange among scientists and decisionmakers to facilitate the adaptive governance of marine resources: A review of knowledge and research needs [18]. Schultz, Lisen (2015) promoted adaptive governance, ecosystem management, and natural capital [19]. Wyborn, Carina (2015) featured co-productive governance: A relational framework for adaptive governance [20]. Gomez-Baggethun, Erik (2015) built in markets we trust? Setting the boundaries of Market-Based Instruments in ecosystem services governance [21].
The most important character of quantum mechanics is entanglement. Ioppolo, Giuseppe (2016) modelled sustainable local development and environmental governance: A Strategic Planning Experience [22]. Bennett, Nathan James (2016) depicted using perceptions as evidence to improve conservation and environmental management [23].

1.2 Organization of the Article
First of all, some preliminaries are introduced in Section 2. In Section 3, a brief review of classical game is given. In Section 4, some equivalent definitions of quantum game is introduced; in Section 4.2, Not only the quantities$D$, D^{μ} , and D^{γ} are given,but

also give the proof of the corresponding inequalities in Theorem 1.1; in addition, in Section 4.2, the remaining quantities D^{σ}, D^{α}, and D^{β} are introduced and the proof of Theorem1.1 is concluded. Throughout Section 4, some example calculations are given to demonstrate the proposed games. In section 5, additional constructions are the main idea : Section 5.1 a partial equivalence between quantum game and rank-one quantum games is [Cooney *et al*. 2011]; in Section 5.2, the rest part of Theorem 1.2 that was not already built in the examples from Section 4 is demonstrated; in Section 5.3, Not only do we study the family (P_i), but slso prove Theorem 1.3.In section 6,some brief explanations which connect our results with standard are involved, but, the results around Grothendieck's inequality in the theory of operator spaces is most important. In section 7, directions for future work are recommoned.

2. Quantum Game Model in China Haze Governance

As mentioned earlier, to a large extent owing to their prefect connection to semidefinite programming and Grothendieck's inequality. Quantum game are easily understood. Unfortunately, it is their simple structure that somewhat limit the kind of behaviors that they can exhibit. For example, as described earlier, in contrast to unentangled strategies, entanglement can offer only a relatively modest advantage. In addition, using a maximally entangled state of relatively small dimension, one is able to achieve the best chance of winning.

the family (∂_i). For any $i \geq 1$, let ∂_i be the quantum game where the players are sent one of the two states

$$|F_1\rangle = \frac{1}{\sqrt{2}}|0\rangle|0\rangle - \frac{1}{\sqrt{2i}}\sum_{k=1}^{i}|k\rangle|k\rangle \quad \text{and} \quad |F_0\rangle = \frac{1}{\sqrt{2i}}\sum_{k=1}^{i}|k\rangle|k\rangle + \frac{1}{\sqrt{2}}|0\rangle|0\rangle,$$

each choose with possibility 1/2 by referee and are required to produce answers with even parity if the state is $|F_0\rangle$, and odd parity if it is $|F_1\rangle$. Orthogonal as the the two states are, how well the players can perform in the game is not a priori clear: how to distinguishcan $|F_0\rangle$ and $|F_1\rangle$ *locally*? Interestingly, the reply to this question crucially rely on the resources vested for the players.

Theorem 1.1. There exists a polynomial-time algorithm, inputs an explicit description of a quantum game χ, outputs two numbers $D^{\gamma}(\chi)$ and $D^{\beta}(\chi)$ such that

$$D(\chi) \leq D^{\alpha}(\chi) \leq D^{\gamma}(\chi) \leq 2\sqrt{2}D(\chi) \quad \text{and} \quad D^{\sigma}(\chi) \leq D^{\beta}(\chi) \leq 2D^{\sigma}(\chi)$$

In analogy with the classical setting, according to Theorem 1.2, one may take it for grounded that the inequalities $D^{\sigma}(\chi) \leq D^{\beta}(\chi)$ and $D^{\alpha}(\chi) \leq D^{\gamma}(\chi)$ in Equation (1) is equalities in fact aspect. Nevertheless, our second family of games, the games (P_i), the properties which are summarized in the following theorem, suggests that equalities do *not* hold in general: in some games, D^{β} is strictly greater than D^{σ} and D^{γ} is strictly greater than D^{α}.

THEOREM 1.3. There exists a family of games (P_i) for which the following hold:

$$\frac{3}{5} = D^{\beta}(P_1) = D^{\gamma}(P_1) > D^{\sigma}(P_1) \geq D^{\alpha}(P_1) > D(P_1) = D^{\mu}(P_1) = \frac{2}{5}$$

and for all $i \geq 1$

$$D^{\mu}(P_i) = D(P_i) = \frac{i+1}{2i+1}D^{\gamma}(P_i) \tag{1}$$

This family of games is linked to the CAR algebra [Bratelli and Robinson 1987, Section 5.2]. As for an unimportant scaling, as presented earlier, the game H1, palying a particularly significant role, can be described concretely as follows. firstly two distinct integers $j < m \in \{1,2,3\}$ are picked by the referee uniformly at random, then one of the two states $\frac{1}{\sqrt{2}}(|j\rangle \pm i|m\rangle)$, again uniformly at random, is sent to each player (thus the probability of each of the four possible combinations is 1 /4). Only if either $a \otimes b = 1$ on condition that they were both sent "+" or "—" states or $a \otimes b = 0$ in the case that they were sent a "+" and a "—" state. the players' answers $a, b \in \{0,1\}$ are accepted by the referee.

3. Characteristics of quantum game

For an integer *i*, we use the notation [*i*] to denote the set {1,...,*i*}. For $x \in R$, we let $sign(\rho) = \rho / |\rho|$ if $\rho \neq 0$, and sign(0) = 1. If $\rho = (\rho_k) \in R^i$ or C^n we let $\|\rho\|_\infty := \max_{k \in [i]} |\rho_k|$. For vectors ρ , θ , μ_i, we define their inner producrt $\langle \rho, \theta \rangle = \sum_i \overline{\rho_i \theta_i}$ and the norm $\|\rho\| = \langle \rho, \rho \rangle^{1/2}$.

Matrices and norms. P_{ω_1} , P_{ω_2} represents the finite dimensional Hilbert space. $\delta(P_{\omega_1}, P_{\omega_2})$ is the set of linear operators from P_{ω_1} to P_{ω_2}, and $\delta(P_{\omega_1}) = \delta(P_{\omega_1}, P_{\omega_1})$. $P(P_{\omega_1})$ is the set of Hermitian operators on P_{ω_1}, and Obs(P_{ω_1}) is the set of observables, that is, Hermitian matrices whose eigenvalues are in {-1, 1}. $\lambda_i(K)$ are used to denote $i \times i$ matrices over a field *K*, and $\lambda_i = \lambda_i(\mu)$. For $\omega_1 \in \delta(P_{\omega_1}, P_{\omega_1}^{'})$, $\|\omega_1\|_\infty$ is its operator norm (i.e., largest singular value) and $\|\omega_1\|_1 := Tr\sqrt{\omega_1 + \omega_1}$ its Schatten 1-norm, it is also known as the standard in quantum information.When the operator ACTS on a tensor product of Hilbert space, we do not always specify the ordering of the product of the tensor product.For example, if $\mu \in \delta(P_A \otimes P_{\omega_2}, P_A^{'} \otimes P_{\omega_2}^{'})$, $\omega_1 \in \delta(P_{\omega_1}, P_{\omega_1}^{'})$, and $\omega_2 \in \delta(P_{\omega_2}, P_{\omega_2}^{'})$, we may depict indifferently $Tr(\mu^+.(\omega_1 \otimes \omega_2))$ or $Tr(\mu^+.(\omega_2 \otimes \omega_1))$ for the same expression (rigorously, the former).

A *classical game* χ of size *n* is specified by n^2 real coefficients $R = (R_{s,t})_{s,t \in [k]}$ satisfying the normalization condition $\sum_{s,t=1}^{i} |R_{s,t}| = 1$. The rules of the game are as follows. The referee chooses a pair of integers $(s,t) \in [i]^2$ according to the distribution $\{\pi(s,t) = | R_{s,t|}\}$, and sends *s* to the first player, Alice, and *t* to the second player, Bob. After receiving their respective questions, the players have one every answer $a, b \in \{0,1\}$. The referee accepts the players' answers if and only if $(-1)^{a \oplus b} = sign(R_{s,t})$. Note that the player who sends a uniform random answer will be accepted as a $1/2\chi$. The deviation of $D(\chi)$ is defined as twice the maximum success probability of any player, and the probability of success of the random strategy (in this case, 1/2) can be formally expressed as

$$D(\chi) = D(R(\chi)) := \max_{\rho_k, \theta_k \in \{\pm 1\}} \left| \sum_{s,t} R_{s,t} \rho_s \theta_t \right| \tag{2}$$

Notice that the maximum value on the right side can be equivalent over all $\rho, \theta \in R^i$ such that $\|\rho_\infty\|, \|\theta_\infty\| \leq 1$ (instead of over all $\rho, \theta \in \{-1,,1\}^i$): The maximum value will always be at an extreme value.

4. Examples and Discussions

4.1 Rank-One Quantum Game Example

Example 4.13 (*Entangled Bias of the Games* (∂_i)).For any *i*, there will be a show for the game ∂_i , with probability arbitrarily close to 1, it can be won and provided that the players sharing an entangled state in large enough dimension is allowed. From the beginning, in Example 4.5, without any entanglement, ∂_1 still can be won with probability 1. To succeed in the game ∂_i for universal *n*, in order to reduce to the case *i* = 1, the players have to use a specific entangled state. *embezzlement states*[10] falls in the family for any *d* ≥ 1 this state is defined as

$$|\Gamma_d\rangle := \frac{1}{\sqrt{d}} \sum_{j=1}^{d} (|i+1\rangle |i+1\rangle)^{\otimes j} \otimes |F_i^\alpha\rangle^{\otimes (d-j)}.$$

which as a bipartite state $|\Gamma_d\rangle \in (\mu^{i+1})^{\otimes d} \otimes (\mu^{i+1})^{\otimes d}$ *is easy to find out* with each register in the natural way,through associating one half of each of the states $\| i+1\rangle |i+1\rangle$ or $|F_i^\alpha\rangle$ Take the following strategy for the players in ∂_i into consideration, defined for any integer *d*. Their private registers in state | ∂_d ⟩ is initialized by the players. After their respective message register is received,

which is controlled on the message register not being in state $|0\rangle$, They are applied to the unitary transformation corresponding to the left cyclic shift of the d + 1 copies of μ^{i+1} in their possession, which leads to the transformation
$(|0\rangle|0\rangle)\otimes|\Gamma_d\rangle \mapsto (|0\rangle|0\rangle)\otimes|\Gamma_d\rangle,$

$$|F_i^{\alpha}\rangle\otimes|\Gamma_d\rangle \mapsto |i+1\rangle|i+1\rangle\otimes(\frac{1}{\sqrt{d}}\sum_{j=1}^{d-1}(|i+1\rangle|i+1\rangle)^{\otimes j}\otimes|F_i^{\alpha}\rangle^{\otimes(d-j)}.$$

Since the right state has an inner product 1 - *O(1/d)* with $|\Gamma_d\rangle$, not only after the cyclic shift but also up to a partial unitary mapping $|i+1\rangle \mapsto |1\rangle$ The player's message register is in a state close to the game ∂_1. After that, they could apply their great strategy to ∂_1, in which case people could verify that their probability of being 1-*O*(1/d) in ∂_i .
In contrast to the untangling situation, in formula (14), relax the upper bound (14), and in most cases, all complex matrices with operator norm will not change the definition of the bias, as shown below.

Formally, an arbitrary (finite-dimensional) Hilbert space *V specifies* a rank-one game $\hat{\chi}$ of size *i*, which corresponds to the umpire's private space and two unit vectors $|w\rangle,|l\rangle \in \mu^i\otimes\mu^i\otimes V$. The game is as follows. The first ready state \ ^) in the three register, λ_{ω_1}, λ_{ω_2}, V corresponds to the space μ^i, respectively. Register λ_{ω_1} is sent to Alice and register λ_{ω_2} is sent to Bob. The players can apply arbitrary unitary operators *U*, *V* on their respective message registers and on their own private spaces P_{ω_1}, P_{ω_2}, which can be initialized in an arbitrary state $|F\rangle \in P_{\omega_1}\otimes P_{\omega_2}$. Registers λ_{ω_1}, λ_{ω_2} will be sent back to the referee by players, and the referee is the person who performs a *rank-one* measurement $\{P^{\phi}, Id-P^{\phi}\}$,where $P^{\phi}=|l\rangle\langle l|$, in the three registers that the referee owns. If the result of " ϕ " is obtained, the referee will accept it or reject it.

Given a rank-one quantum game $\hat{\chi}=(|w\rangle,|l\rangle)$, think of it the (not necessarily Hermitian) matrix. $\hat{\lambda}=\hat{\chi}(\hat{\lambda}) := Tr|w\rangle\langle l|$. The maximum acceptance probability of any entangled players in $\hat{\chi}$ equals is obvious.

$$D^{vc1}(\hat{\chi}) = D^{vc1}(\hat{\lambda}(\hat{\chi})) := \sup_{\substack{P_{\omega_1},P_{\omega_2},|F\rangle,|r\rangle\in P_{\omega_1}\otimes P_{\omega_2},\\ U\in\delta(\mu^i\otimes P_{\omega_1}),V\in\delta(\mu^i\otimes P_{\omega_2})}} |Tr((U\otimes V)(\hat{\lambda}\otimes|F\rangle\langle r|))|^2$$

where all finite-dimensional Hilbert spaces P_{ω_1} and P_{ω_2} take over the supremum, unit not only vectors, $|F\rangle,|r\rangle\in P_{\omega_1}\otimes P_{\omega_2}$ but also $U\in\delta(\mu^i\otimes P_{\omega_1}), V\in\delta(\mu^i\otimes P_{\omega_1})$ of an operator norm at most 1.

Lemma 5.3. $\hat{\chi}$ can be considered as an arbitrary rank-one quantum game. Then a quantum game χ is existence such as $D^{\sigma}(\chi)=(D^{vc1}(\hat{\chi}))^{1/2}$.

Proof. Make $\hat{\chi}=(|w\rangle,|l\rangle)$ be a rank-one quantum game of size *i*. $\hat{\chi}$ is associated to the following quantum game χ of size 2*i*. In χ, one of two possible states are prepared by the referee

$$|F_{\pm}\rangle := \frac{1}{\sqrt{2}}(|0\rangle_{\lambda_{w_1}}|0\rangle_{\lambda_{w_2}}|l\rangle_{\lambda_{w_1}\lambda_{w_2}V} \pm |1\rangle_{\lambda_{w_1}}|1\rangle_{\lambda_{w_2}}|l\rangle_{\lambda_{w_1}\lambda_{w_2}V}),$$

each with probability 1/2. notice that the message registers in χ have one less qubit each than those in *G*. the players' respective message registers are sent by the referee, and their answers are accepted only when 0 is the parity in the case and $|F_+\rangle$, was the prepared state and $|F_-\rangle$ was the case in 1. The corresponding competition matrix is

$$\lambda = \lambda(\chi) = \frac{1}{2}(|00\rangle\langle 11|\otimes Tr_V(|w\rangle\langle l|) + |11\rangle\langle 00|\otimes Tr_V(|l\rangle\langle w|))$$
$$= \frac{1}{2}(|00\rangle\langle 11|\otimes\hat{\lambda} + |11\rangle\langle 00|\otimes\hat{\lambda}^{+})$$

According to the type, $\lambda = \hat{\lambda}(\hat{\chi})$ is the matrix associated to $\hat{\chi}$.
$D^{\sigma}(\chi) \le (D^{vc1}(\hat{\chi}))^{1/2}$ will be shown first.Make not only $q > 0$ but also $(\omega_1, \omega_2 \mid F\rangle)$ a strategy which is for the players of χ achieving a bias at least $(1-q)D^{\sigma}(\chi)$. Define $U := (\langle 1 | \otimes Id)\omega_1(\langle 0 | \otimes Id)$, $V := (\langle 1 | \otimes Id)\omega_2(\langle 0 | \otimes Id)$ and $| r\rangle := | F\rangle$.After not only *U* but *V* both have a norm at most 1.As a result, a valid assignment to the right-hand side of (20) is formed by the quadruple $(U, V, | r\rangle, | F\rangle)$.

$$D^{vc1}(\hat{\chi}) \ge | Tr((U \otimes V)(\hat{\lambda} \otimes | F\rangle\langle F |)) |^2 = | Tr((\omega_1 \otimes \omega_2)(\lambda \otimes | F\rangle\langle F |)) |^2 \ge ((1-q)D^{\sigma}(\chi))^2$$

Is the resulting value, where the proof of this direction of the inequality by letting $q \to 0$ is concluded.
$D^{\sigma}(\chi) \ge (D^{vc1}(\hat{\chi}))^{1/2}$ are remained to show. Make no only $U, V, | F\rangle$ but also $| r\rangle$ achieve a value at least $(1-q)D^{vc1}(\hat{\chi})$ in the Equation (20)'s right-hand side. Assume that if there are no loss of generality that both $| F\rangle, | r\rangle \in P \otimes P$ for some finite-dimensional *P*. notice that through changing the phase of $| F\rangle$, Let's say that the expression in the absolute value is true and non-negative and this is feasible. While Equation (20)is allowed to use different state $| F\rangle$, $| r\rangle$, the definition which is of the bias of a quantum game (Equation (14)) not only reflects their operational interpretation but also calls for both states to be identical. To accommodate for this, a strategy for the players in χ which is based on the use of an embezzlement state can be constructed. Letting *d* be a dimension parameter is d ($d \to \infty$ is taken the limitation), it is defined as

$$| \Gamma_d \rangle := \frac{1}{\sqrt{D}} \sum_{j=1}^{d} b^j \, | F\rangle^{\otimes j} \otimes | r\rangle^{\otimes (d-j)}$$

where $b = \frac{\langle F | r\rangle}{|\langle F | r\rangle|}$ is considered to be non-orthogonal states, otherwise $b = 1$, and the appropriate normalization factor is $d \le D \le d^2$. The players share not only $| F\rangle$ but also $| \Gamma_d \rangle$, *d* + 1 copies of *P* altogether will be given to each player. For the first player's *d* + 1 copies the unitary transformation is $\tilde{V}$ which is corresponding to a cyclic shift, As same as the first play, $\tilde{V}$ is for the second player, accompanied by multiplication by a phase $\tilde{b}$, so that

$$\tilde{U} \otimes \tilde{V} : | \Phi\rangle \, | \Gamma_d \rangle \mapsto | F\rangle \, | \Gamma_d \rangle,$$

where $| \Gamma_d \rangle = (1/\sqrt{D}) \sum_{j=0}^{d-1} b^j \, | F\rangle^{\otimes j} \otimes | r\rangle^{\otimes (d-j)}$. Notice that $\left\| | \Gamma_d \rangle - \langle \tilde{\Gamma}_d \right\|^2 \le 4/d$, thus $\mathrm{Re}\langle \Gamma_d \mid \Gamma_d \rangle \ge 1 - 2/d$. Let $\omega_1 := | 0\rangle\langle 1 | \otimes (U.\tilde{U}) + | 1\rangle\langle 0 | \otimes (U.\tilde{U})^+$ and $\omega_2 := | 0\rangle\langle 1 | \otimes (V.\tilde{V}) + | 1\rangle\langle 0 | \otimes (V.\tilde{V})^+$.

Both ω_1 and ω_2 can be verified and have a norm at most 1, and as the same as Claim 4.14,

$$\begin{aligned} D^{\sigma}(\chi) &\ge | Tr((\omega_1 \otimes \omega_2)(\lambda \otimes | r\rangle\langle r | \otimes | \Gamma_d \rangle\langle \Gamma_d |)) | \\ &= \frac{1}{2} | \langle w | \langle r | \langle \Gamma_d \, | U.\tilde{U} \otimes V.\tilde{V} \, | l\rangle \, | r\rangle \, | \Gamma_d \rangle + \langle l \, | \langle r \, | \langle \Gamma_d \, | .\tilde{U}^+ U^+ \otimes \tilde{V}^+ V^+ \, | w\rangle \, | r\rangle \, | \Gamma_d \rangle \\ &= | \mathrm{Re}(\langle w | \, r \, | \Gamma_d \, | U \otimes V \, | l\rangle \, | r\rangle \, | .\langle \Gamma_d \, | \Gamma_d \rangle) | \\ &\ge (1 - \frac{2}{d})((1-q)D^{vc1}(\hat{\chi}))^{1/2} \end{aligned}$$

Not only take the limit as $q \to 0$ but $d \to \infty$ ends the second part's proof of the lemma.

4.2 Entangled strategy for quantum game example

For the sake of the interested reader, this section is concluded with briefly explaining how results appearing in the literature deduce Theorem 6.3. In the left-hand side of Equation (40) is the supremum. The goal is to be reformulated in an equivalent form which can be direct compared with statements that appear in the operator space literature. What we define in the first and main step is

$$\psi = \left\{ \left(\overrightarrow{M_R}, \overrightarrow{M_\mu}, \overrightarrow{N_R}, \overrightarrow{N_\mu} \right) \in \left(\overrightarrow{Mat_d} \left(\mu^i \right) \right)^4 \middle| d \geq 1, \overrightarrow{M_R} \otimes \overrightarrow{N_\mu} = \overrightarrow{M_\mu} \otimes \overrightarrow{M_R} \right\}$$

and recall that elements of ψ satisfying Equation (41)take over the supremum in the left-hand side of Equation (40). Denote that supremum by sup ψ . think over the set

$$\psi' = \left\{ \left(\overrightarrow{M_R}, \overrightarrow{M_\mu}, \overrightarrow{N_R}, \overrightarrow{N_\mu} \right) \in \left(\overrightarrow{Mat_d} \left(\mu^i \right) \right)^4 \middle| d \geq 1, \quad \forall k \in \{1,...,d\}, \left(\left(\overrightarrow{M_R} \right)_k, \left(\overrightarrow{N_R} \right)_k \right) and \left(\left(\overrightarrow{M_\mu} \right)_k, \left(\overrightarrow{N_\mu} \right)_k \right) \right\}$$

{are non negatively proportional},

Using non negatively proportional we claim that the two associated $2i^2$-dimensional complex vectors suit that one is a result of the other with a non negative real number (or, equivalently, that they are based on the similar closed ray). It is not hard to check that $\psi' \subseteq \psi$; hence, if we let $\sup_{\psi'}$ indicate the modified supremum, as appearing in the left-hand side of Equation (40) but now replaced by the elements of $\psi\,^r$ which fit Equation (41), we get that $\sup_{\psi'} \leq \sup_{\psi}$. this is declared, in fact, an equality.

,*d>i is supposed at f*irst, as otherwise the i is used to append zero coordinates. Now, Fact 6.4 is applied to ω_{11} and ω_{12} and U_1, U_2 , R, D_1, D_2 is regarded as the resulting matrices. The assumption implies that

$$R\left(D_1 O_{o\times(d-o)}\right)\left(\omega_{21}U_1\right)^+ = R\left(D_2 O_{o\times(d-o)}\right)\left(\omega_{22}U_2\right)^+ ,$$

which, due to *R* is of full column rank, implies

$$\left(\omega_{21}U_1\right)\begin{pmatrix} D_1 \\ O_{(d-o)\times o} \end{pmatrix} = \left(\omega_{22}U_2\right)\begin{pmatrix} D_2 \\ O_{(d-o)\times o} \end{pmatrix},$$

Together with Equation (42), we get

$$\begin{pmatrix} \omega_{12}U_2 \\ \omega_{21}U_1 \end{pmatrix}\begin{pmatrix} D_1 \\ O_{(d-o)\times o} \end{pmatrix} = \begin{pmatrix} \omega_{11}U_1 \\ \omega_{22}U_2 \end{pmatrix}\begin{pmatrix} D_2 \\ O_{(d-o)\times o} \end{pmatrix}$$

therefore, since either D_1 or D_2 are zero at the same location, $\left(\left(\omega_{11}U_1\right)_i, \left(\omega_{22}U_2\right)_i\right)$ are non negatively proportional when $k \in \{1,...,o\}$. For *k>o*, $\left(\left(\omega_{12}U_2\right)_i, \left(\omega_{21}U_1\right)_i\right)$ are observed as zero. Thus, by taking $V_1 = U_1U_1', V_2 = U_2U'_2$,the proof can be completed, for the i some tries U'_1, U_2' defined as

$$U_1' = \begin{pmatrix} Id & 0 & o \\ 0 & Id & o \end{pmatrix}, \quad U_2' = \begin{pmatrix} Id & 0 & o \\ 0 & 0 & Id \end{pmatrix} \quad \omega_{11} \; \omega_{12}$$

where *o, d* — *o, d* — *o* are the dimensions of the three column blocks, respectively, *o, d* — *o* are those of the row blocks.

In order to show that $\sup_{\psi'} = \sup_{\psi}$ as claimed, any tuple $\left(\overrightarrow{M_R}, \overrightarrow{M_\mu}, \overrightarrow{N_R}, \overrightarrow{N_\mu}\right) \in \psi \;\psi$ is taken and Claim 6.5 is applied with ω_{11}, ω_{12} , ω_{21} ,and take ω_{22} to be the $i^2 \times d$ matrices whose rows include the vector entries of $\overrightarrow{M_R}, \overrightarrow{M_\mu}, \overrightarrow{N_\mu},$ and, $\overrightarrow{N_R}$, respectively. The reason why the condition $\omega_{1i}\;\omega_{2i}$ = $\omega_{12}\;\omega_{22}$ holds is that it is equivalent to $\overrightarrow{M_R} \otimes \overrightarrow{N_\mu} = \overrightarrow{M_\mu} \otimes \overrightarrow{N_R}$. There exist two isometries, which is shown by the claim. therefore if one is applied to the vector entries of $\overrightarrow{M_R}$ and of N_μ and the other is appied to the vector entries of $\overrightarrow{M_\mu}$ and of $\overrightarrow{N_R}$, then in ψ ' *is* the resulting tuple $\left(\overrightarrow{M_R}, \overrightarrow{M_\mu}, \overrightarrow{N_R}, \overrightarrow{N_\mu}\right)$. Since applying isometries is all we did, not only does the new tuple achieve the same target function but also satisfies the constraint (Equation (41)). Taking the supremumover ψ ' is once again equivalent to the original form is clear.

In the second step, consider the set ψ '' defined like ψ '', in addition, instead of a nonnegative one, *positive* proportionality was required (in other word, one can be obtained from another by multiplication with a positive number). since the goal function and the constraint (Equation (41)) are continual, It is obvious that taking the supremum over ψ '' is again equivalent to the original form.

Let λ be the matrix which is associated to the game ∂_i. the singular value decomposition $\lambda = \frac{1}{2}|00\rangle\langle F_i^{\alpha}| + \frac{1}{2}|F_i^{\alpha}\rangle\langle 00|$ as in the proof of Claim 5.1 is used to get the rank-one quantum game $\hat{\partial}_i$ defined by

$|w\rangle = \frac{1}{\sqrt{2}}(|00\rangle|0\rangle + |F_i^{\alpha}\rangle|1|\rangle)$. and $|l\rangle = \frac{1}{\sqrt{2}}(|F_i^{\alpha}\rangle|0\rangle + |00\rangle|1\rangle)$

the fact is $D^{\sigma}(\partial_i) = 1$. Moreover, as we will show in Claim 5.6, In order for them to succeed with probability approaching 1 in ∂_i, it is *necessary to use* unbounded amount of entanglement between the players. In contrast, using a maximally entangled state of dimension *i*. It is not hard to let the rank-one game $\hat{\partial}_i$ be won perfectly. Hence, the example also demonstrates that it is not completely avoidable for the use of arbitrarily high-dimensional embezzlement states in the transformation $\hat{\chi} \to \chi$ which is given in the proof of Lemma 5.3.In this section, the proof of Theorem 1.2 is completed. What were shown in Examples 4.5 and 4.9 is the first sequence of equalities. in Example 4.13,there show the equality $D^{\sigma}(\partial_i) = 1$ and the general inequalities $D^{\sigma} \le D^{\beta}$ and $D^{\beta} \le 1$ are followed by $D^{\beta}(\partial_i) = 1$,which is proven in Theorem 4.19. Therefore the “moreover” part remains to prove, namely, the probability of the perfect winning will only be achieved in the limit of infinite entanglement.

CLAIM 5.6. Let *q*>0 be small enough, *i*≥ 2 and d be an integer. On assumption that $(\omega_1, \omega_2, |F\rangle)$,where $|F\rangle \in \mu^d \otimes \mu^d$, is regarded as a strategy prepared for the players in the game ∂_i, achieving a bias at least 1 - *q*. Then $d \ge i^{\mu/\sqrt{q}}$, where μ >0 is a general constant.

The fact that $(\omega_1, \omega_2, |F\rangle)$ achieves a bias at least $1-q$ in ∂_i implies, by definition,

$$Tr_{\mu^{i+1}\otimes\mu^{i+1}}(\langle F|(\omega_1 \otimes \omega_2)(\lambda(\partial_i) \otimes Id)|F\rangle) |=| \mathrm{Re}(\langle 00|\rangle F|(\omega_1 \otimes \omega_2)|F_i^{\alpha}\rangle|F\rangle) | \ge (1-q) .$$

the following fact is used, implicit in Leung *et al*. [2008, Section 3]. (Its proof is generated by the Fuchs-van de Graaf inequalities, which let the fidelity be related to the trace norm, and Fannes’ inequality, which offers a lower bound on the trace distance between two density matrices according to their different von Neumann entropies.)

Fact 5.7. Let *i*, *d* be integers, $U, V \in \delta(\mu^i \otimes \mu^d)$ arbitrary operators of a norm at most 1, and $|E\rangle \in \mu^i \otimes \mu^i \; |F\rangle \in \mu^d \otimes \mu^d$ of unit norm. Let *S* be the von Neumann entropy of the reduced density of $|E\rangle$ on any of the two subsystems, and presume $\varphi \ge 1$.
Then

$$1 - |\langle E|\langle F|U \otimes V|0^i 0^i\rangle|F\rangle|^2 \ge \min\left\{\frac{1}{4e^2}, \frac{\varphi^2}{16\log^2(3d)}\right\}$$

the fact is applied to *U* = ω_1, *V* = ω_2 *and* $|E\rangle = |F_i^{\alpha}\rangle$, by using that the reduced density of the maximally entangled state $|F_i^{\alpha}\rangle$ has von Neumann entropy log *i*, we find that the tactics $(\omega_1, \omega_2, |F\rangle)$ must contented $q \ge \mu^2 \log^2(i)/\log^2(d)$ for some universal constant μ > 0.

5. Conclusions and Further Directions

This article focused on debataly three-party games’ simplest type is called Quantum game. These are three-party, one-round games, and the referee’s conduct of the games is limited: each player offers him only a one-bit answer, and he is restricted to make his final decision (accept/reject) based on the parity of the two bits alone.

To count the players’ success through their *bias* is habitual, defined as twice the difference between the players’ success probability and their success probability if they answered all questions randomly in the background of Quantum game. In order to improve the bias over players restricted to sharing a assured type of nonlocal resource cause different quantities of interest. The *unentangled bias* $D(\chi)$ is the correspondence of the largest possible bias achievable by players restricted to not using any entanglement at all. The *entangled bias* $D^{\sigma}(\chi)$ is the correspondence of players who may share an arbitrary entangled state. What’s more, to consider the *maximally entangled bias,* $D^{\alpha} * (\chi)$, which corresponds to players restricted to sharing a maximally entangled state of arbitrary dimension will be interesting.

Acknowledgments
This research was supported by the National Natural Science Foundation of China (No. 71471102), and Yichang University Applied Basic Research Project in China (Grant No. A17-302-a13).

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