



Computer simulation of the mathematical models for the determination of the uplift resistance of a gas pipeline

Dr. Mathew Shadrack Uzoma¹, Dr. OMO Etebu²

^{1,2} Department of Mechanical Engineering, University of Port Harcourt, Port Harcourt, Rivers State, Nigeria

Abstract

Mathematical models for the determination of the uplift resistance of gas pipelines network system have been developed. The computer simulation of the mathematical models is the bane of this work. The paramount parameters in view to be determined by computer simulated algorithms are: optimal wall thickness, the restoring moment, pipe deflection and pipe support spacing for a laid pipeline to avoid buckling of the pipeline subject to the different loading conditions. At nominal pipe wall thickness of 11cm, the simulated optimal wall thickness is 17cm. The simulation results confirmed the deflection of the pipe as -2.698m, the restoring force as $-3.33 \times 10^{17} \text{N}$. Subject to these findings, three pipe supports are required to bear restoring load of $1.11 \times 10^{17} \text{N}$ at each support.

Keywords: uplift resistance, earth mass centroid, pipe thickness, maximum deflection, support reaction, buckling load

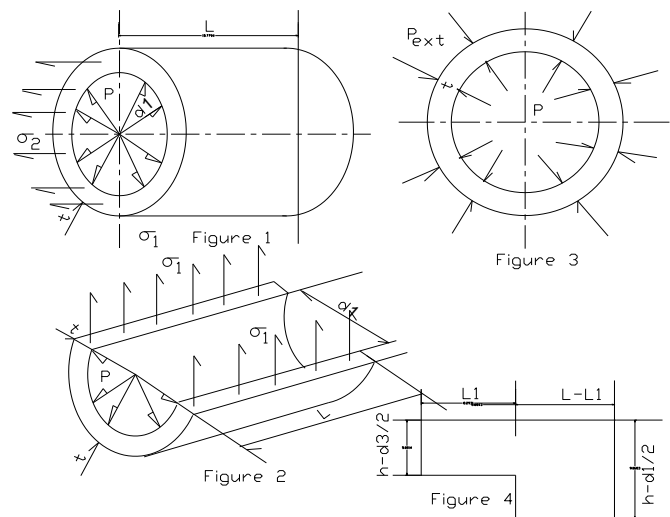
1. Introduction

Gas pipelines assets and facilities are expensive capital intensive production items designed to have a long service life. Failure of these vital production assets might have serious economic and environmental implications. There are published works on uplift mechanisms of buried pipeline caused by creep, earth movement, inadvertently resulting in infilling of soil at the base of the pipe [1, 2, 3, 4, 5]. This situation will result in pressure build up at the base of the pipe, leading to gradual drift of the pipe to the earth surface. The worst case situation is the buckling of the pipe.

Oil and gas pipeline mechanical design concepts based on ANSI/ASME set of standard codes adequately specified design equations for pipe wall thickness, flow velocity, compressors, valves, fittings and flanges design, pipe support spacing, flow density and pipe internal diameter. The applicable set of standard codes are: ANSI/ASME B31, 3, B31.4, B31.8 and API RPI4C [6, 7, 8, 9].

The easiest, safest and the most efficient and economical means of transportation or transmission of fluid be it liquid or gas is through interconnected pipes generally referred to as pipeline network system. In gas transmission system, the pipeline in three subcategories namely: the gathering pipeline (field pipeline), the main trunk line and the service or distribution line. By virtue of the terrain transversed by the pipeline, weight of the pipes and that of the fluid being conducted, the pipeline might be subject to diverse stresses and subsequently buckling. This necessitates the need for model formulation to determine the safe pipe wall thickness, the uplift resistance of the pipe that will prompt accurate pipe support spacing on the basis of deflection of the pipe. Computer simulation of the developed models has been incorporated in this work to enable the determination of the controlling parameters for the uplift resistance. This approach would culminate in improved design, higher transmission efficiency and lesser cost of installation of gas pipeline assets

and facilities. The geometric representation of the pipeline system for stresses and uplift resistance computations are as in Figures 1 to 5.



2. Purpose and Significance

Uplift mechanisms of a buried pipe or the prevailing conditions militating for the upward drift of a buried pipe had been reviewed. Oil and gas pipelines mechanical design concepts were reviewed in all its details. This work strongly rooted on the formulation of mathematical models for the uplift resistance of laid buried pipeline, is to enable the determination of optimal pipe wall thickness, support reactions and pipe deflection. The pipeline irrespective of the length is imagined as supported at the ends. It is believed if the pipe deflection is known to a measure of accuracy pipe supports can be installed along the pipeline to annul the deflection to avoid catastrophic failure due to buckling. The computer simulation of mathematical models would enable the determination of those paramount parameters such as

supports reactions, optimum wall thickness, bending moment, pipe deflection and support placement by computational approach. This way our future oil and gas pipelines would meet the design and operation standard required for optimal performance.

3. Mathematical Models for Computer Simulation

The geometric representation of the forces and stresses acting on the system are as presented in Figures 1 to 5.

Circumferential stress on the structure is expressed as:

$$\sigma_1 = \frac{Pd_1}{2t} \quad (1)$$

Longitudinal stress is given as:

$$\sigma_2 = \frac{Pd_1}{4t} \quad (2)$$

Pipe mean diameter

$$d_m = \frac{d_1 + d_1 + t}{2} = d_1 + t/2 \quad (3)$$

The expression for the radial stress goes thus;

$$\sigma_3 = \frac{Pd_1 - P_{atm}(d_1 + t)}{d_1 + t/2} \quad (4)$$

Where:

P —average stream pressure (N/m²)

σ_1 —hoop stress or circumferential stress (N/m²)

L —pipe length (m)

t, \bar{t} —pipe thickness (m)

d_1 —pipe internal diameter (m)

The resultant external loads on the structure is expressed as:

$$P_{ext} = P_{atm} + P_{earth} + P_c + P_w$$

$$P_{atm} = 1bar$$

$$P_{earth} = \rho_s gh$$

$$P_w = \phi P_g \text{ at } (33^\circ C)$$

If the pipe is encased in a concrete of thickness t_1 , the load intensity is expressed as,

$$P_c = \frac{F_c}{A_c} = \frac{m_c g}{A_c} = \frac{\rho_c V_c g}{A_c} = \frac{\rho_c t_1 g (d_2 + d_1)}{2d_1}$$

$$A_c = \pi d_2 L$$

$$V_c = \frac{\pi(d_2^2 - d_1^2)L}{4} = \frac{\pi(d_2 + d_1)(d_2 - d_1)L}{4} = \pi L(d_2 - d_1) \\ d_2 - d_1 = 2t$$

Where:

σ_2 —longitudinal stress (N/m²)

d_m —pipe mean diameter (m)

P_{ext} —resultant external loads on the pipe (N/m²)

σ_3 —radial stress (N/m²)

P_{ext} —resultant external loads on the pipe (N/m²)

P_{atm} —atmospheric pressure (N/m²)

P_{earth} —pressure due to overlying earth (N/m²)

P_c —pressure due to the weight of the concrete coating (N/m²)

P_w —pressure due to atmospheric water vapor water (N/m²)

F_c —Force exerted by the casing (N)

m_c —mass of the concrete casing (kg)

t_1 —casing thickness (m)

ρ_c —density of casing material (kg/m³)

V_c —volume of concrete (m³)

A_c —inner curved surface area of the concrete coating (m²)

d_2 —pipe outer diameter (m)

V_c —volume of concrete (m³)

V_p —pipe volume (m³)

V_e —earth volume (m³)

V_g —volume of gas in the pipeline (m³)

Subject to the tri-axial stress condition, the maximum shear stress is the greatest of the three values.

$$\tau_{1max} = \left| \frac{\sigma_1 - \sigma_2}{2} \right| \text{ or } \left| \frac{\sigma_2 - \sigma_3}{2} \right| \text{ or } \left| \frac{\sigma_3 - \sigma_1}{2} \right| \\ \therefore \tau_{1max} = \left| \frac{\sigma_3 - \sigma_1}{2} \right| \quad (5)$$

Under uni-axial stress condition, $\sigma_2, \sigma_3 = 0$

$$\tau_{2max} = \frac{\sigma_1}{2} \quad (6)$$

Temperature stress in the system is expressed as;

$$\sigma_T = E \varepsilon = E \frac{\Delta L}{L} = E \frac{\alpha L \Delta T}{L} = E \alpha \Delta T \quad (7)$$

On the basis of these analyses the overall induced maximum

shear stress in the pipe can be expressed as:

$$\begin{aligned}\tau_{\max} &= \tau_{1\max} - \tau_{2\max} \\ &= \frac{\sigma_3 - \sigma_1}{2} - \frac{\sigma_T}{2}\end{aligned}\quad (8)$$

Applying failure (yielding) analysis known as maximum shear stress theory.

$$\sigma_1 - \sigma_3 - \sigma_T = \frac{\sigma_{ypt}}{FS}\quad (9)$$

σ_{ypt} is the yield point of the pipe material under uni-axial tension test.

Where:

σ_3 --radial stress (N/m²)

$\tau_{1\max}$ --maximum shear stress under tri-axial stress condition (N/m²)

τ_{\max} --induced maximum shear stress in the pipe (N/m²)

σ_T --temperature stress (N/m²)

ε --poisson ratio

E—Young's modulus of elasticity for the pipe (N/m²)

α —linear expansivity of the pipe material (°C)

ΔT —temperature difference between the pipe gas and the environment (°C)

ΔL —change in length of the pipe (m)

$$\begin{aligned}\frac{Pd_1}{2t} - \frac{Pd_1 - P_{ext}}{d_1 + t/2} - \sigma_T &= \frac{\sigma_{ypt}}{FS} \\ t^2 \left[-2P_{ext} - \sigma_T - \frac{\sigma_{ypt}}{FS} \right] + t \left[-1.5Pd_1 + 2P_{ext}d_1 - 2\sigma_Td_1 - \frac{2\sigma_{ypt}}{FS} \right] + Pd_1 &= 0\end{aligned}\quad (10)$$

$$a = -2P_{ext} - \sigma_T - \frac{\sigma_{ypt}}{FS}$$

$$b = -1.5Pd_1 + 2P_{ext}d_1 - 2\sigma_Td_1 - \frac{2\sigma_{ypt}}{FS}$$

$$c = Pd_1$$

$$\bar{t} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\quad (11)$$

\bar{t} is the design wall thickness of the pipe for safe operation with due regard to the required operating conditions.

The centroid of the earth mass above the pipe is expressed as:

$$\bar{x} = \frac{(L - L_1) \left(h - \frac{d_2}{2} \right) \left(\frac{L - L_1}{2} \right) + L_1 \left(h - \frac{d_3}{2} \right) \frac{2L - L_1}{2}}{(L - L_1) \left(h - \frac{d_2}{2} \right) + L_1 \left(h - \frac{d_3}{2} \right)}\quad (12)$$

Load Components

(i) Pipe Weight

$$W_p = m_p g = \rho_p V_p g = \frac{\pi L_3 \rho_p g (d_1^2 - d_2^2)}{4}\quad (13)$$

(ii) Concrete Weight

$$W_c = m_c g = \rho_c V_c g = \frac{\pi L_1 \rho_c g (d_3^2 - d_2^2)}{4}\quad (14)$$

(iii) Earth Mass Weight

$$\begin{aligned}W_e &= m_e g = \rho_e V_e g \\ &= \pi \rho_e g \left[h^2 L - \frac{d_2^2 L}{4} - \frac{(d_3^2 - d_2^2) L_1}{4} \right] \\ V_e &= \pi \left[h^2 L - \frac{d_2^2 L}{4} - \frac{(d_3^2 - d_2^2) L_1}{4} \right]\end{aligned}\quad (15)$$

Weight of gas inside the pipe

$$\begin{aligned}m_n &= \frac{PV_g}{ZRT} \\ R &= \sum \frac{m_i}{m_H} R_i, \quad m_i = n_i M_i \\ W_n &= m_n g\end{aligned}\quad (16)$$

Where:

m_p —mass of pipe (Kg)

m_c —mass of concrete (Kg)

m_e —mass of overlying earth (Kg)

m_H —mass of hydrocarbon constituents (Kg)

m_i —mass fraction of the constituents (Kg)

n_i —number of moles of the constituents

M_i —molar mass of the constituents (Kg)

R —average gas constant for the constituents (J/KgK)

R_i —gas constant for the constituents (J.KgK)

g —gravitational acceleration (m/s²)

W_p —weight of pipe (N)

W_c —weight of concrete (N)

W_e —weight of overlying earth mass (N)

W_g —weight of pipe (N)
 F —restoring force (N)
 R_1, R_2 —reactions at the supports (N)

Applying Macaulay's method ^[10], for the moment distribution along the pipeline,

$$EI \frac{d^2 y}{dx^2} = M \quad (17)$$

$$= R_1 x - W_c(x - L_1/2) - W_p(x - L/2) - W_n(x - L_1/2) - W_e[x - (L - \bar{x})]$$

$$EI \frac{dy}{dx} = \frac{R_1 x^2}{2} - W_c \left(\frac{x^2}{2} - \frac{L_1 x}{2} \right) - (W_p + W_n) \left(\frac{x^2}{2} - \frac{Lx}{2} \right) - W_e \left(\frac{x^2}{2} - L_2 x - \bar{x}x \right) + A_1 \quad (18)$$

$$EI y = \frac{R_1 x^3}{6} - W_c \left(\frac{x^3}{6} - \frac{L_1 x^2}{4} \right) - (W_p + W_n) \left(\frac{x^3}{6} - \frac{Lx^2}{4} \right) - W_e \left(\frac{x^3}{6} - \frac{L_2 x^2}{2} + \bar{x}x \right) + A_1 x + B_1 \quad (19)$$

Applying the boundary conditions when $x=0, y=0$ and when $x=L, y=0$
 $B_1 = 0$

$$A_1 = W_c \left[\frac{L^2}{6} - \frac{L_1 L}{4} \right] - \frac{L^2}{12} [W_p + W_n] + W_e \left(\frac{\bar{x}L}{2} - \frac{L^2}{3} \right) - \frac{R_1 L^2}{6}$$

The reaction R_1 , and R_2 are given as,

$$R_1 = \frac{W_c \left(\frac{2L - L_1}{2} \right) + (W_p + W_c)L/2 + W_e \bar{x}}{L}$$

$$R_2 = (W_c + W_e + W_p + W_n) - R_1$$

At the point of maximum deflection, $dy/dx=0$, hence,

$$x^2 \left[\frac{R_1}{2} - \frac{W_c}{2} - \left(\frac{W_p + W_n}{2} \right) - \frac{W_e}{2} \right] - x \left[\frac{W_c L_1}{2} + \frac{(W_p + W_n)L}{2} + W_e(L - \bar{x}) \right] + A_1 = 0$$

$$a_1 = \frac{R_1}{2} - \frac{W_c}{2} - \left(\frac{W_p + W_n}{2} \right) - \frac{W_e}{2}$$

$$b_1 = \left[\frac{W_c L_1}{2} + \frac{(W_p + W_n)L}{2} + W_e(L - \bar{x}) \right]$$

$$c_1 = A_1$$

$$\bar{c} = \frac{-b_1 \pm \sqrt{b_1^2 - 4a_1 c_1}}{2a_1} \quad (20)$$

The maximum deflection maximum deflection is expressed as:

$$y = \left[\frac{R_1 \bar{c}^3}{6} - W_c \left(\frac{\bar{c}^2}{6} - \frac{L_1 \bar{c}}{2} \right) - (W_p + W_n) \left(\frac{\bar{c}^3}{6} - \frac{L \bar{c}^2}{4} \right) - W_e \left(\frac{\bar{c}^2}{6} - \frac{L \bar{c}^2}{2} + \bar{x} \bar{c} \right) + A_1 \bar{c} \right] / EI$$

The bending moment at the point of maximum deflection is given by the expression;

$$M = R_1 \bar{c} - W_c \left(\bar{c} - \frac{L_1}{2} \right) - (W_p + W_n) \left(\bar{c} - L/2 \right) - W_e \left[\bar{c} - (L - \bar{x}) \right]$$

The restoring force, F , at point if maximum deflection is gives as

$$Fy = M, \quad F = M / y$$

This is the required force to prevent buckling of the pipeline subject to the loading conditions. This force can be offered by providing additional support at point \bar{C} or overdesigning the pipe to be of such thickness to withstand the buckling load F . The buckling load F is referred as the uplift resistance of the pipe.

4. Computer Simulation for the Determination of the uplift Resistance

The programming language is MATLAB ^[11].

% Computer simulation for the uplift resistance of gas pipelines network

% System (Production data of shell Petroleum Development Corporation, August 2008)

% 1/ Simulation for pipes pipes optimum wall thickness

% Initializing

% Upstream pressure at SOKU, P1(N/m2)

P1=81*10^5;

fprintf('%20.6f\n', P1)

% Downstream pressure at bonny (N/M2)

P2=63*10^5;

% Average stream pressure, P (N/m2)

P=(2/3)*(P1^3-P2^3)/(P1^2-P2^2);

% Atmospheric Pressure, Patm (N/m2)

Patm=1.03*19^5;

% Soil Density (SANDY LOAM DS (kg/m3)

DS=1940;

% Density of concrete, DC (kg/m3)

DC=2310;

% MILD STEEL PIPE DENSITY, DP (kg/m3)

DP=7820;

% Pipe nominal diameter (36") (m)

d1=0.9144;

% Pipe thickness, t1 (m)

t1=0.11;

% Pipe outer diameter, d2 (m)

d2=d1+2*t1;

% Thickness of concrete casing, t2 (m)

t2=0.015;

% Temperature drop along line, DT (k)

DT=246;

% Outer diameter of concrete casing, d3 (m)

d3=d2+2*t2;

% Pipe Length, L (m)

L=116000;

L1=(2/3)*L;

L2=L-L1;

% Burial depth of the pipe from the centre line of pipe, h (m)

h=100;

% Young's modulus of elasticity for mild steel, E (N/m2)
E=213*10^9;

% Factor of safety for the system, FS
FS=1.5;

% Linear Expansivity for Mild Steel, A (/K)
A=4.03*10^(-8);

% Yield stress for mild steel, YS, (N/m2)
YS=260*10^6;

% Bulk flow temperature, T (K)
T=313;

% Temperature difference, DT (K)
TD=246;

% Flow compressibility factor, Z
Z=1.241;

% Gravitational acceleration, g (m/s2)
g=9.81;

% General gas constant, R0 (J/kgK)
R0=8314;

% Circumferential stress, CS1 (N/m2)
CS1=(P*d1)/(2*t1);

% Longitudinal stress, (N/m2)
CS2=(P*d1)/(4*t1);

% Bearing pressure due to the earth mass

% Volume of earth mass, Ve, (m3)
Ve=((22/7)*(h^2*L)-((d2^2-d1^2)*L)/4-((d3^2-d2^2)*L1)/4);

% Weight of earth mass, We (N)
We=DS*Ve*g;
disp(' We ')
fprintf('%20.6f\n', We)

% Bearing pressure due to the earth mass
Pext=We/((22/7)*d3*L1+d2*(L-L1));
disp(' Ptex ')
fprintf('%20.6f\n', Pext)

% Radial Stress, CS3 (N/m2)
CS3=(-(P*d1)+Pext*(d1+t1))/(d1+t1);
disp(' CS1 CS2 CS3')
fprintf('%20.6f\n', CS1, CS2, CS3)

% TEMPERATURE STRESS, CT (N/m2)
CT=E*A*DT;

% To calculate the design wall thickness of the pipe for safe operation
a=-2*Pext*d1-CT-(YS/FS);
b=-1.5*P*d1+2*Pext*d1-2*CT*d1-(YS/FS);
c=P*d1;

% To calculate the design wall thickness of the pipe for safe operation, th(m)

th1=(-b-(-b^2-(4*a*c))^(0.5))/(2*a);
th2=(-b-(-b^2+(4*a*c))^(0.5))/(2*a);
disp('th1 th2')
fprintf('%12.6f\n',th1, th2)

% Shell average gas composition

C1=0.869859; C2=0.054574; C3=0.020709;
IC4=0.004507; NC4=0.006309;
IC5=0.002178; NC5=0.000787; C6=0.004627;
N2=0.000598; CO2=0.034843;

% Molecular mass of the gaseous mixture

M1=16; M2=30; M3=44; IM4=54; NM4=54; IM5=72;
NM5=72; M6=86; MN2=28; MCO2=44;

% Average molecular mass of the gas, M (kg/mol)

M=C1*M1+C2*M2+C3*M3+IC4*IM4+NC4*NM4+IC5*IM5+NC5*NM5+C6*M6+N2*MN2+CO2*MCO2;
disp(' M ')
fprintf('%20.6f\n', M)

% Average gas constant, R (J/kgK)

R=R0/M;
disp(' R ')
fprintf('%20.6f\n', R)

% Average flow pressure, P (N/m2)

P=92/3*(P1^3-P2^3)/(P1^2-P2^2);

% Pipe weight, Wp (N)

Wp=((22/7)*(d2^2-d1^2)*DP*L*g)/4;

% Concrete weight, Wc, (N)

Wc=((22/7)*(d3^2-d2^2)*DC*L1*g)/4;

% Volume of gas in the pipeline, Vg (m3)

Vg=((22/7)*d1^2)*L;

% Weight of gas in the pipeline

Wg=(P*Vg*g)/(Z*R*T);
disp(' Wp Wc We Wg')
fprintf('%12.6f\n',Wp, Wc,We,Wg)

% Simulation for the uplift resistance of the pipeline, F (N)

% To determine x bar (C1)

R1=((Wc*(2*L-L1)/2)+((Wp+Wg)*L)/2+(We*C1))/L;
C1=((L-L1)*(h-(d1/2))*((L-L1)/2)+(L1*(h-(d3/2))*((2*L-L1)/2)))/((L-L1)*(h-(d2/2))+L1*(h-(d3/2)));
A1=Wc*((L^2/6)-((L1*L)/4))-
(Wp+Wg)*(L^2/12)+We*((C1*L)/2)-(L^2/3))-
((R1*L^2)/6);
R2=(We+Wp+Wc+Wg)-R1;

% Determination of X-coordinate point x (m)

a1=R1/2-Wc/2-((Wp+Wg)/2)-We/2;
b1=((Wc*L1)/2)-(Wp+Wg)*(L/2)+We*(L-C1);
c1=A1;
x1=(-b1-(-b1^2-(4*a1*c1))^(0.5))/(2*a1);

```
x2=(-b1-(+b1^2-(4*a1*c1))^(0.5))/(2*a1);
disp(' x1, x2')
fprintf('%12.6f\n',x1,x2)
```

```
% To determine the system moment of inertia, I (m4)
I=((22/7)*((16*h^4)-d1^4))/64;
disp(' I ')
fprintf('%12.6f\n',I)
```

```
% To determine the maximum deflection of the system, ymax
ymax1=(((R1*x1^3)/6)-Wc*(((x1^3)/6)-((L1*x1^2)/2))-
(Wp+Wg)*(((x1^3)/6)-((L*x1^2)/4))-We*(((x1^3)/6)-
((L*x1^2)/2)+(A1*x1))/(E*I*10^8);
disp(' ymax1 ')
fprintf('%12.6f\n',ymax1)
```

```
ymax2=(((R1*x2^3)/6)-Wc*(((x2^3)/6)-((L1*x2^2)/2))-
(Wp+Wg)*(((x2^3)/6)-((L*x2^2)/4))-We*(((x2^3)/6)-
((L*x2^2)/2)+(A1*x2))/(E*I*10^8);
disp(' ymax2 ')
fprintf('%12.6f\n',ymax2)
```

```
% The bending moment at the point of maximum deflection,
MB, (Nm)
MB1=R1*x1-Wc*(x1-(L1/2))-(Wp+Wg)*(x1-(L/2))
```

```
We*(x1-(L-C1));
disp(' MB1 ymax1 F1 ')
F1=(MB1/ymax1);
fprintf('%20.6f\n',MB1,ymax1,F1)
MB2=R1*x2-Wc*(x2-(L1/2))-(Wp+Wg)*(x2-(L/2))-
```

```
We*(x2-(L-C1));
disp(' MB2 ymax2 F2')
F2=(MB2/ymax2);
fprintf('%20.6f\n',MB2,ymax2,F2)
```

5. Model Applications

The concept of uplift resistance can be applied to oil or gas pipelines network system to provide additional supports to prevent buckling of the pipeline. It is a well known fact that oil and gas pipelines are laid over a distance of thousands of kilometer and they are expensive production assets that last for long period of time. Hence the structure should be well designed with adequate support spacing.

6. Recommendation

The simulated computational results should be applied to physical gas pipeline at the design and installation stage. This is to enhance the service and operational life of this expensive production asset.

7. Conclusion

Computer simulation of gas pipeline uplift resistance has been developed for the determination of safe optimum pipe wall thickness, uplift resistance, and the point of maximum pipe deflection to enable proper spacing of pipe supports. This is to avoid failure of the pipe by the reason of fatigue and bending

tresses that will ultimately result in the buckling of the pipe under service conditions.

8. References

1. Guijt J. Uplift buckling of offshore pipeline – Overview and introduction. Paper OTC 6487, Offshore Technology Conference, (1990) Houston, Texas.
2. Finch M. Uplift buckling and floatation of rigid pipeline. The influence of recent geotechnical research on the current state of the art. Paper OTC 10713, Offshore Technology Conference, (1999) Houston, Texas.
3. Dickin EA. Uplift resistance of buried pipeline in sand. Soil and Foundations. 1994; 342:41-48.
4. Chin EL, Craig WH, Crukshank M. Uplift resistance of pipeline buried in cohesionless soil." Proc. 6th Int. Conf. on Physical Modelling In Geotechnics. Ng., Zhang, and Wang, eds., Taylor & Francis Group, London. 2006; 1:723-728.
5. Schainee PEL, Zorm NF, Shotmann GJM. Soil response for pipeline upheaval analysis: Full-scale laboratory test and modeling. Proc. 22nd Annual Offshore Technology, 1990.
6. ANSI/ASME Standard B31.1: For Chemical Plant and Production Refinery Piping, (2002) New York City: ANSI/ASME.
7. ANSI/ASME Standard B31.4: Standard for Liquid Transportation Systems for Hydrocarbon, liquid petroleum Gas, Anhydrous Ammonia and Alcohols, (2002) New York City: ANSI/ASME.
8. ANSI/ASME Standard B31.8: Standard for Gas Transmission and Distribution Piping Systems, (1999) New York City: ANSI/ASME.
9. API RPI4C: Recommended Practice for Analysis, Design, Installation and Testing of Basic Surface Safety Systems for Offshore Production Facilities, seventh edition, (2001) Washinton, DC.
10. William A Nash. Schaum's Outline Series, Theory and Problems of Strength of Materials, 2ND Edition, McGraw-Hill Book Company, 1977. ISBN: 07 084366 X
11. William J. Palm III. Introduction to Matlab 7 for Engineers, McGraw Hill International Edition, (2005), ISBN: 007-123262-1, New York, NY.